

A Direct Sampling Method Based on the Green's Function for Time-Dependent Inverse Scattering Problems

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Abstract. This paper is devoted to numerical simulation of time domain inverse acoustic scattering problems with a point-like scatterer, multiple point-like scatterers or normal size scatterers. Using Green's functions and time convolution, we propose direct sampling methods to reconstruct the location of the scatterer. The methods involve only integral calculus without solving any equations and are easy to implement. Numerical experiments show the effectiveness and robustness of the methods.

AMS subject classifications: 65M32

Key words: Inverse scattering problem, time-dependent, direct sampling method, point-like scatterer, Green's function.

1. Introduction

The inverse scattering problems are widely used in geophysical exploration, sonar detection, non-destructive testing and medical imaging [12, 27, 31]. In the past decades, frequency domain analysis is one of the main topics in the inverse acoustic scattering studies [2, 3, 19, 23]. Nevertheless, since in practical applications the dynamic scattered data is easier to obtain, the time domain inverse scattering attracted attention of numerous researchers lately [6, 25].

This paper deals with a simple direct sampling method for inverse scattering problems in the time domain. Sampling methods play an important role in solving inverse problems. Various sampling methods, such as the linear sampling method [9, 11, 22], the direct sampling method [17], the reverse time migration method [10] and the factorization method [18], have been studied in the past decades. The basic idea of the direct sampling methods is to construct indicator functions that have different performances inside and outside the area of the scatterer. Compared to the iterative methods, sampling methods do not require a priori information of the geometry and physical properties of unknown target and can be easily implemented.

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For frequency domain inverse scattering problems, the linear sampling method was first proposed by Colton and Kirsch [11]. A multilevel linear sampling method and its enhancement are proposed in [21] and [20], respectively. Based on the far field operator and the far field pattern of the scattered field, a direct sampling method to reconstruct the scatterers in the frequency domain is proposed in [24]. In [5, 30], simultaneous reconstructions of an obstacle and its excitation sources in frequency domain are considered.

For time domain inverse scattering problems, the efficiency of the linear sampling method was shown by Chen *et al.* [9]. In [7, 29], the linear sampling method is used to solve the time domain inverse scattering problems in a locally perturbed half-plane and the time domain inverse scattering problems with cracks, respectively. The time domain linear sampling method is also considered to reconstruct an obstacle with Robin or Neumann boundary condition from near field measurements [16] and reconstruct penetrable obstacles [15]. Based on the migration method, effective sampling schemes to reconstruct small and extended scatterers from knowledge of time-dependent scattered data are proposed in [14].

The reconstruction of point-like scatterers has been studied by image-based direct methods in [17]. Note that the Green's function plays an important role in the analysis of the inverse problems of mathematical physics — cf. Refs. [4, 6, 13, 26]. In [8], the approximations of the solution to the forward scattering problem are analyzed by utilizing the Green's function and the retarded single-layer potential, and an indicator function based on an approximate solution is defined to reconstruct the location of a single point-like scatterer. Unfortunately, the proposed method in [8] is not effective to reconstruct multiple point-like scatterers or normal size scatterers. In this paper, a novel direct sampling method is proposed to reconstruct point-like scatterer based on the approximate solution to the forward scattering problem and the application of the time convolution. Moreover, modified methods are defined, which are feasible to reconstruct a single point-like scatterer, multiple point-like scatterers and normal size scatterers.

The rest of the paper is organized as follows. A brief introduction of forward and inverse scattering problems under consideration is given in Section 2. In Section 3, a novel direct sampling method is proposed and the theoretical analysis of the method for reconstructing a point-like scatterer is provided. Moreover, two modified methods are also considered in this section. Numerical examples in two and three dimensional spaces presented in Section 4, show the effectiveness and robustness of the methods. Finally, the paper is concluded in Section 5.

2. Problem Setting

Let the scatterer is located in a bounded region $D \in \mathbb{R}^3$ and $c > 0$ be the sound speed of the homogeneous background medium in $\mathbb{R}^3 \setminus \overline{D}$. The Green's function of the D'Alembert operator $c^{-2}\partial_{tt} - \Delta$ is

$$G(x, t; y) = \frac{\delta(t - c^{-1}|x - y|)}{4\pi|x - y|}, \quad x, y \in \mathbb{R}^3, \quad x \neq y, \quad t \in \mathbb{R},$$

where Δ is the Laplacian in $\mathbb{R}^3$ and $\delta(t)$ is the Dirac delta distribution.