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A Fast Collocation Method with Inscribed Polygonal Approximation of the Peridynamic Neighborhood for Bond-Based Linear Viscoelastic Peridynamic Models

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Abstract. The peridynamic (PD) theory is a reformulation of the classical theory of continuum solid mechanics, which yields an integro-differential equation that does not involve spatial derivatives of the displacement field. Therefore, it is particularly suitable for the description of cracks and their evolution in materials. Due to the nonlocal property of PD models, classical numerical methods usually generate dense stiffness matrices which require $\mathscr{O}(N^3)$ computational work and $\mathscr{O}(N^2)$ memory, where N is the number of spatial unknowns. In this paper, we develop a fast collocation method for a bond-based linear viscoelastic peridynamic model in two space dimensions by exploring the structure of the stiffness matrix. The method has a computational work count of $\mathscr{O}(N\log N)$ per Krylov subspace iteration and $\mathscr{O}(N)$ memory requirements. In our model, the peridynamic neighborhood is a two-dimensional circular domain, so that one has to deal with the intersection of such circles and rectangular meshes used in the collocation method. We employ an inscribed polygon to approximate the peridynamic neighborhood. This improves the numerical accuracy of solution and reduces programming complexity. Numerical results show the utility of the proposed method.

AMS subject classifications: 65M50, 65M70

Key words: Nonlocal model, viscoelastic peridynamic model, fast collocation method, inscribed polygonal approximation.

1. Introduction

The classical continuum theory assumes that material points interact with their immediate neighbors only. This is a local theory which generates mathematical models that can be described by partial differential equations. Therefore, the basic mathematical structure of the formulation breaks down whenever a crack appears in a body. Nonlocal theories in continuum mechanics have been studied at least since 1967, cf. [1, 2, 20–22, 39]. The PD

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theory [28, 29] is a nonlocal theory mathematically compatible with describing the formation of discontinuities such as cracks and their evolution in materials. These advantages attracted considerable attention, and the peridynamic theory has been successfully used in the study of failure and damage in composite laminates [18, 42], crack propagation and branching [17, 19], crack nucleation [32], impact damage [3, 37], crystal plasticity [34], damage in concrete [14], etc.

The peridynamic models for elastic material have been widely developed due to its profound theoretical connotation and abundant application in science and engineering [4, 11, 12, 14, 46]. In addition to elastic materials, there are materials categorized as viscoelastic. They exhibit characteristics that are both solid- and fluid-like [13, 43]. Because of the fluid-like behavior, the viscoelastic materials exhibit a time-dependent constitutive law and are categorized as rate-type materials. Thus similar to the plastic materials, the viscoelastic materials belong to the class of materials with memory — viz. its current state of stress does not only depend on the current deformation, as in elastic materials, but also on deformations experienced in the past. Unlike the plastic materials with permanent memory, the viscoelastic materials have so-call fading memory. The later means that the deformations occurring in the distant past have less effect on the current state of the stress. To investigate more complicated material behavior such as size-dependent superelastic effect observed in experiments of single crystal Cu-Al-Ni shape memory alloys, researchers have considered accounting for nonlocal effects to go beyond local viscoelasticity [25]. A nonlocal formulation called peridynamic, has been successfully employed to describe the behavior of viscoelastic materials [30, 33, 40].

To solve PD models, a large variety of numerical methods have been proposed so far. For instance, easily implementable collocation and meshfree methods can be applied directly to the strong form of the PD models [23, 26, 27, 31]. On the other hand, Galerkin finite element methods are used to discretize the weak form of the PD models and enjoy high convergence orders [6, 24, 35, 45]. However, PD models present new challenges not encountered in classical continuum solid mechanical models — viz.

(1) The singularity of influence function has a great effect on the accuracy and convergence of numerical methods for the peridynamic models.

(2) Due to nonlocal properties of PD models, classical numerical methods often produce dense stiffness matrices. Hence, the direct solutions of these linear systems require $\mathscr{O}(N^3)$ computational work per time step and $\mathscr{O}(N^2)$ of storage memory.

(3) Peridynamic neighborhood is ubiquitously chosen to be a circular domain in two space dimensions, so that one has to deal with the intersection of such circles and rectangular meshes.

In this paper, we develop a fast and accurate collocation method for a bond-based linear viscoelastic PD model. The method has $\mathscr{O}(N\log N)$ computational work per Krylov subspace iteration and an optimal-order memory requirement of $\mathscr{O}(N)$. This is achieved without resorting to any lossy compression by exploring structure of the stiffness matrices