East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.2023-317.170324

Vol. **15**, No. 2, pp. 392-414
May 2025

A New Framework of Troubled-Cell Indicator on General Meshes for DG Methods Using Simple Statistical Analysis Techniques

Hongqiang Zhu[1], Haiyun Wang[2], Zhihuan Wang[1], Rumeng Shao[1] and Zhen Gao[2,*]

[1]*School of Natural Sciences, Nanjing University of Posts and Telecommunications, Nanjing 210023, P.R. China.*
[2]*School of Mathematical Sciences, Ocean University of China, Qingdao 266100, P.R. China.*

Received 12 December 2023; Accepted (in revised version) 17 March 2024.

Abstract. An accurate, efficient and robust troubled-cell indicator (TCI) is always required by the discontinuous Galerkin methods for solving the nonlinear system of hyperbolic conservation laws. In this paper, we propose a new framework of TCI on general meshes using simple statistical analysis techniques which is easy to implement and efficient to run. In the framework troubled-cell indication is performed in each local stencil. We first pick the local stencils that contain the troubled cells via the range of the normalized troubled-cell indication values, and then detect the troubled cells in each picked stencil by a simple separation approach. The new TCI framework is implemented on several types of meshes with three indication variables. Numerical experiments conducted through simulating several classical shocked flows demonstrate that the new framework can capture the locations of shock waves accurately and lead to essentially nonoscillatory solutions. The results also verify the robustness of the framework.

AMS subject classifications: 65M60, 35L60, 35L65, 35L67

Key words: Troubled-cell indicator, limiter, discontinuous Galerkin method, statistical analysis.

1. Introduction

Since the end of the last century, the Runge-Kutta discontinuous Galerkin (RKDG) finite element method [6] has been applied in different research fields [1,4,28] due to high order of accuracy, flexibility in handling complicated geometries, easy h-p adaptation and efficiency of parallel implementation. When it is used to solve the nonlinear system of hyperbolic conservation laws in an essentially nonoscillatory way, the nonlinear limiter is

*Corresponding author. *Email addresses:* zhuhq@njupt.edu.cn (H. Zhu), wanghaiyun@stu.ouc.edu.cn (H. Wang), 2502798805@qq.com (Z. Wang), 1021082409@njupt.edu.cn (R. Shao), zhengao@ouc.edu.cn (Z. Gao)

a necessary part for detecting the discontinuities and controlling the spurious oscillations — e.g. the minmod type TVB limiter [6], the moment based limiters [2,3], the monotonicity-preserving (MP) limiter [22], the modified MP limiter [18], and the subcell limiters [7,10]. A limiter first detects the troubled cells in the discontinuous regions and then reconstructs the numerical solution in those cells to control the spurious oscillations. According to Qiu and Shu [15], the limiter usually consists of two parts — viz. a troubled-cell indicator (TCI) and a solution reconstruction method. An ideal TCI should detect the troubled cells accurately in an efficient way without problem-dependent parameters. Otherwise, the RKDG methods may produce spurious oscillations if too few cells are detected, or burden extra CPU time with possible loss of numerical resolutions if too many cells are detected.

A systematic review of limiters before the year of 2014 is available in [7]. In 2016, Vuik and Ryan [23] provided an automatic parameter selection strategy based on the Tukey's boxplot method for various troubled-cell indication variables. Fu and Shu [9] introduced a TCI that depends only on the data from the target cell and its immediate neighbors. This work was generalized to h-adaptive meshes by Zhu *et al*. [31]. Machine learning techniques have been popularly used in the last decade due to rapid development of high performance computing techniques and computer hardware. Artificial neural networks with supervised deep learning were introduced to design TCIs [8,16,17,21]. In our previous works [26,35], we proposed a framework of TCI based on an unsupervised machine learning technique — i.e. the K-means clustering. However, K-means clustering is known to be NP-hard.

In this study, a new framework of TCI based on simple statistical analysis techniques is proposed for the RKDG method for solving the nonlinear system of hyperbolic conservation laws. It only involves two simple statistical concepts — viz. the statistical normalization and range of data. In this TCI framework, the computational cells are grouped into local stencils and troubled-cell indication is conducted stencil by stencil. We first pick the local stencils that contain the troubled cells by using the range of the normalized data, and then detect the troubled cells in each picked stencil by employing a simple separation approach. The new TCI framework is easy to implement and efficient to run. It can work on very general meshes with various indication variables. Generally speaking, the parameters in the framework are not sensitive to the test problems, indication variables and meshes. We carry out numerical experiments through simulating several classical shocked flows and test three indication variables on several types of meshes, including one with hanging nodes. The results demonstrate that the new TCI can capture the locations of shock waves or large gradients accurately and lead to essentially nonoscillatory solutions.

The remainder of this paper is organized as follows. We give a brief review of the RKDG method in Section 2. The new TCI framework is proposed in Section 3, including a detailed description of the algorithm and the indication variables. The numerical results are discussed in Section 4 and the concluding remarks are given in Section 5.

2. RKDG Method

We briefly explain the RKDG method using the initial-value problem of the hyperbolic systems of conservation laws