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A First-Order Image Super-Resolution Model Promoting Sharp Boundaries and Suppressing the Staircase Effect

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Abstract. We propose a new non-convex first-order variational model for the image super-resolution problem. The model employs a recently developed regularizer that has proven to be effective in image restoration. Due to this regularizer, the salient feature of our model lies in the fact it can construct sharp edges in those generated super-resolution images from lower-resolution ones. Moreover, it also helps suppress the staircase effect. The maximum-minimum principle is proved, which indicates that there is no need to impose hard constraints on the objective function. Alternating direction method of multipliers with spectral penalty selection is utilized to minimize the associated functional. Cartoon and real gray and color images are tested to demonstrate the features of our model to show the comparison with state-of-the-art image super-resolution techniques.

AMS subject classifications: 65M32, 94A08, 65K10

Key words: Image super-resolution, variational model, alternating direction method of multipliers.

1. Introduction

Image super-resolution is a process to estimate a high-resolution image from one or multiple low-resolution images, and it has a wide range of applications in medical imaging, high definition television, satellite imaging, synthetic aperture radar, and so on. In this paper, we mainly consider the challenging single-frame super-resolution problem, that is, we want to construct a high-resolution image by up-sampling a single image. In general, it is of course impossible to recover fine-scale details that are missing from low-resolution images.

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We can only hope to construct some very specific structures, or produce visually pleasing high-frequency textures. The super-resolution restoration problem is ill-conditioned since the low-resolution image we get is obtained by the original image through a convolution operator and a down-sampling operator. Quite a few models and algorithms have been proposed to deal with this problem: interpolation based methods [2, 15, 19, 32], least squares based methods [3,14], variational methods [12,26], machine learning methods [11, 13, 23, 37, 43], and other statistics based methods. In order to make the method more effective, Fang *et al.* [16] added the image prior knowledge into the model, proposed a new edge-guided image restoration framework and developed an accurate Soft-edge assisted network (SeaNet) for image super-resolution. To solve the problem of computing and memory cost caused by deep learning, Gao *et al.* [17] proposed a lightweight bimodal network (LBNet) for single image super-resolution.

In order to avoid the ringing effect and smearing effect, Marquina and Osher proposed a variational model that employs total variation as the regularizer for image super-resolution [26]. For simplicity, this model is abbreviated as the MO model. Just as the well-known ROF model [34, 44], this model suffers from the staircase effect and it always tends to underestimate the amplitudes of signal discontinuities, that is, the MO model is subject to the loss of image contrast [5, 8, 27]. To fix the staircase phenomenon, many higher-order variational models have been developed for image restoration, where different regularizers were proposed, including total generalized variation, Euler's elastica, nonlinear fourth-order diffusive term, and so on [6, 9, 38, 45]. Even though these models have proved efficient for staircase reduction, higher-order models tends to blur image edges and texture, and one has to deal with more complicated models than the total variation based ones. Lower-order variational models were also proposed for staircase reduction [31, 35, 47]. To ameliorate the loss of contrast, many non-convex models were developed, including Nikonova's nonconvex regularized least-squares [28,29], Wu *et al.* [40] truncated regularization, logarithmic TV (logTV) norm [31,36], minimax-concave penalty (MCP) [35], capped$-\ell^1$ (CL1) penalty [25], TV^q (TV^q) norm [21], and Cauchy-based (Cauchy) penalty [22].

Recently, a novel first-order regularizer was proposed for image restoration with the aim to avoid the staircase effect while also keeping image contrasts — cf. [48]. Unlike the Huber model, this regularizer has a lower growth rate than total variation over regions with relatively large image gradient in order to keep image contrast. The model has been applied for image denoising and deblurring. The experiments in [48] show that the model could efficiently remove noise while also preserving image contrast and suppressing the staircase phenomenon. More importantly, when applied for deblurring problems, this model renders deblurred images with more clean boundaries than the ROF model, which is mainly due to the fact that the new regularizer allows larger jumps than total variation. In this paper, we introduce the regularizer from [48] into the image super-resolution problem with the aim to preserve sharp image jumps and eliminate the staircase effect. Specifically, we note that some models use the characteristic function to impose hard constraints on the objective function [1], so we give the maximum-minimum principle which indicates that this characteristic function is unnecessary.