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The Application of Deterministic and Random SEQIR Models in COVID-19 Pandemic and the Study of Threshold Behavior

Yaxin Zhou[1] and Daqing Jiang[1,2,*]

[1]*College of Science, China University of Petroleum (East China), Qingdao 266580, P.R. China.*
[2]*Pakistan Academy of Sciences, Islamabad, Pakistan.*

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Abstract. Up to now, COVID-19 caused by SARS-CoV-2 is still widely spreading. Most patients have a good prognosis, with some critically ill patients dying. This paper considers the SEQIR COVID-19 model with standard incidence. Based on the characteristics of the model, we study the content of threshold behavior in deterministic and stochastic systems. We can first perform dimensionality reduction on the model due to the fact that the reduced model has the same stability as the equilibrium point of the original model. We first express the local stability of boundary equilibrium points for deterministic system after dimension reduction with the method of Lyapunov functions. After considering the perturbation of logarithmic Ornstein-Uhlenbeck processes, we study the existence and uniqueness of positive solutions. Subsequently, the critical value R_0^s related to the basic regeneration number R_0 was obtained. And then, the conditions of R_0^s about the persistence and extinction of the disease is in-depth researched, it is a critical condition. When $R_0^s < 1$, the disease tends to become extinct, while when $R_0^s > 1$, the system exhibits a stationary distribution. And the density function near the positive equilibrium point is described in detail. Finally, our conclusions are well supported through numerical simulation.

AMS subject classifications: 34K25, 34F05, 37A30, 37H30

Key words: Logarithmic Ornstein-Uhlenbeck processes, threshold, local stability, stationary distribution, density function.

1. Introduction

As the COVID-19 entered its fourth year, the monitoring dropped sharply. Although the weekly reported cases and deaths are at their lowest level since the outbreak began, millions of people continue to be infected or reinfected with severe acute respiratory syndrome

*Corresponding author. *Email addresses:* zhouyaxin2020@163.com (Y. Zhou), daqingjiang2010@hotmail.com (D. Jiang)

coronavirus 2, resulting in thousands of deaths per week [34]. The number of secondary infections is still increasing, and even the number of people who have been infected three times is not an exception. The infection report of COVID-19 in May this year in China presented in [3] shows that COVID-19 still has a great impact. The clinical symptoms of most infected people are relatively mild, and only a few people infected with novel coronavirus for the second time may have severe symptoms such as high fever and dyspnea [2]. The public generally expressed that the physical strength and various functions of the body after infection are not as good as before, but the physical health monitoring data are still within the normal range. This phenomenon cannot be explained without experimental data. Therefore, effective control measures in the early stages of the epidemic are still applicable, including maintaining physical distance, good hygiene conditions, good ventilation, mask protection, diagnosis, self isolation of infected individuals and vaccination.

During the epidemic of infectious diseases, epidemiologists, biologists, mathematicians, and others were busy with epidemic prevention work. They established mathematical models based on some data from the early stages of the epidemic outbreak, providing insights on the spread rate and peak prediction of the epidemic, providing important references for relevant departments' epidemic prevention decisions and making due contributions to the fight against the epidemic. Ndairou *et al.* [27] showed that the mathematical models and corresponding theoretical analysis of various infectious diseases established according to the disease transmission mechanism have been very mature, and have played a significant role in the fight against the COVID-19. The accuracy and diversity of data, as well as cross validation of data, are important prerequisites for ensuring accurate prediction of models and evaluating the effectiveness of decision-making basis. Promoting China's experience through data mining and mathematical model comparative analysis to serve international epidemic decision-making and prevention and control. Musa *et al.* [26] states that the mathematical model of the COVID-19 has the effect of raising awareness. Interventions including intensive contact tracing followed by quarantine and isolation are indicated by Tang *et al.* [31], and it estimated the transmission risk of COVID-19 and its impact on public health intervention. Memon *et al.* [22] studied and verified the role of quarantine and isolation as a control strategy for COVID-19 and they theoretically analyzed the model based on the actual data of domestic COVID-19 cases in the country, put forward the conclusion that extend the isolation period as an effective tool to deal with the COVID-19 cases. Anastassopoulou *et al.* [1] analyzed, modeled and predicted the COVID-19 based on data and pointed out that we are in a digital and globalized world, new data and information about the novel corona virus and its epidemic situation can quickly come into sight. But it is still impossible to obtain accurate answers for predicting the dynamics of the epidemic. The uncertainty of existing official data, especially data related to the true baseline number of infections (cases), can lead to ambiguous analysis results and inaccurate predictions of order of magnitude. For more models of COVID-19, see Refs. [10, 28, 29, 35].

As it mentioned in [43], interventions such as media networks have a significant impact on the spread of diseases, people have deepened their understanding of diseases and strengthened their own protective measures. It can also be seen that unknown factors in a random environment can cause significant deviations in the trend of disease changes. Mao