Vol. 5 No. 2 JOURNAL OF COMPUTATIONAL MATHEMATICS April 1987

# SEVEN-DIAGONAL AND NINE-DIAGONAL SIP ALGORITHMS*

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## § 1. Introduction

It is well-known that the SIP method[1,2] is one of the best methods for solving systems of linear algebraic equations, but the convergence and the convergence rate of the method are a difficult problem. Moreover, till now only the case of five-diagonal matrix has been considered. In this paper we present the SIP algorithm for the case of seven-diagonal, nine-diagonal and more-diagonal matrices and consider its convergence and the rate of convergence. when the coefficient matrix is a diagonally dominant matrix.

## § 2. SIP Algorithm

Consider the matrix equation

$$Ax=f, \tag{1}$$

where

$$x=(x_{11}, \cdots, x_{1n}, \cdots, x_{m1}, \cdots, x_{mn})^T, \tag{2}$$

$$f=(f_{11}, \cdots, f_{1n}, \cdots, f_{m1}, \cdots, f_{mn})^T, \tag{3}$$

$$A=\begin{bmatrix} B_1 & C_1 & & & \\ A_2 & B_2 & C_2 & & \\ \cdots & \cdots & \cdots & \cdots & \\ & & & A_m & B_m \end{bmatrix}, \tag{4}$$

$$A_i=\begin{bmatrix} \eta_{i1} & \zeta_{i1} & & & \\ \xi_{i2} & \eta_{i2} & \zeta_{i2} & & \\ \cdots & \cdots & \cdots & \cdots & \\ & & & \xi_{in} & \eta_{in} \end{bmatrix}, \quad B_i=\begin{bmatrix} b_{i1} & c_{i1} & & & \\ a_{i2} & b_{i2} & c_{i2} & & \\ \cdots & \cdots & \cdots & \cdots & \\ & & & a_{in} & b_{in} \end{bmatrix},$$

$$C_i=\begin{bmatrix} \beta_{i1} & \gamma_{i1} & & & \\ \alpha_{i2} & \beta_{i2} & \gamma_{i2} & & \\ \cdots & \cdots & \cdots & \cdots & \\ & & & \alpha_{in} & \beta_{in} \end{bmatrix}. \tag{5}$$

Let

$$L=\begin{bmatrix} L_1 & & & \\ K_2 & \ddots & & \\ & \ddots & \ddots & \\ & & K_m & L_m \end{bmatrix}, \quad U=\begin{bmatrix} U_1 & V_1 & & \\ & \ddots & \ddots & \\ & & \ddots & V_{m-1} \\ & & & U_m \end{bmatrix}, \tag{6}$$

* Received December 4, 1985.

where

$$L_i=\begin{bmatrix} l_{i1} & & & \\ m_{i2} & \ddots & & \\ & \ddots & \ddots & \\ & & m_{in} & l_{in} \end{bmatrix},\quad K_i=\begin{bmatrix} e_{i1} & f_{i1} & & \\ d_{i2} & e_{i2} & f_{i2} & \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \\ & & d_{in} & e_{in} \end{bmatrix},$$

$$U_i=\begin{bmatrix} 1 & u_{i1} & & \\ & \ddots & \ddots & \\ & & \ddots & u_{in-1} \\ & & & 1 \end{bmatrix},\quad V_i=\begin{bmatrix} w_{i1} & t_{i1} & & \\ s_{i2} & w_{i2} & t_{i2} & \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \\ & & s_{in} & w_{in} \end{bmatrix}. \tag{7}$$

Therefore

$$LU=\begin{bmatrix} L_1U_1 & L_1V_1 & & \\ K_2U_1 & K_2V_1+L_2U_2 & L_2V_2 & \\ \cdots & \cdots & \cdots & \\ & & K_mU_{m-1} & K_mV_{m-1}+L_mU_m \end{bmatrix}, \tag{8}$$

where

$$K_iU_{i-1}=\begin{bmatrix} e_{i1} & e_{i1}u_{i-1,1}+f_{i1} & f_{i1}u_{i-1,2} & & \\ d_{i2} & d_{i2}u_{i-1,1}+e_{i2} & e_{i2}u_{i-1,2}+f_{i2} & f_{i2}u_{i-1,3} & \\ \cdots & \cdots & \cdots & \cdots & \\ & & & d_{in} & d_{in}u_{i-1,n-1}+e_n \end{bmatrix},$$

$$L_iV_i=\begin{bmatrix} l_{i1}w_{i1} & l_{i1}t_{i1} & & & \\ m_{i2}w_{i1}+l_{i2}s_{i2} & m_{i2}t_{i1}+l_{i2}w_{i2} & l_{i2}t_{i2} & & \\ m_{i3}s_{i2} & m_{i3}w_{i2}+l_{i3}s_{i3} & m_{i3}t_{i2}+l_{i3}w_{i3} & l_{i3}t_{i3} & \\ \cdots & \cdots & \cdots & \cdots & \\ & & m_{in}s_{in-1} & m_{in}w_{in-1}+l_{in}s_{in} & m_{in}t_{in-1}+l_{in}w_{in} \end{bmatrix},$$

$$K_iV_{i-1}=\begin{bmatrix} e_{i1}w_{i-1,1}+f_{i1}s_{i-1,2} & e_{i1}t_{i-1,1}+f_{i1}w_{i-1,2} & f_{i1}t_{i-1,2} & & & \\ d_{i2}w_{i-1,1}+e_{i2}s_{i-1,2} & d_{i2}t_{i-1,1}+e_{i2}w_{i-1,2}+f_{i2}s_{i-1,3} & e_{i2}t_{i-1,2}+f_{i2}w_{i-1,3} & f_{i2}t_{i-1,3} & & \\ d_{i3}s_{i-1,2} & d_{i3}w_{i-1,2}+e_{i3}s_{i-1,3} & d_{i3}t_{i-1,2}+e_{i3}w_{i-1,3}+f_{i3}s_{i-1,4} & e_{i3}t_{i-1,3}+f_{i3}w_{i-1,4} & f_{i3}t_{i-1,4} & \\ \cdots & \cdots & \cdots & \cdots & & \\ & & d_{in}s_{i-1,n-1} & & d_{in}t_{i-1,n-1}+e_{in}w_{i-1,n} & \\ & & & & & d_{in}w_{i-1,n-1}+e_{in}s_{i-1,n} \end{bmatrix}, \tag{9}$$

$$L_iU_i=\begin{bmatrix} l_{i1} & l_{i1}u_{i1} & & & \\ m_{i2} & m_{i2}u_{i1}+l_{i2} & l_{i2}u_{i2} & & \\ & m_{i3} & m_{i3}u_{i2}+l_{i3} & l_{i3}u_{i3} & \\ & \cdots & \cdots & \cdots & \\ & & & m_{in} & m_{in}u_{i,n-1}+l_{in} \end{bmatrix}.$$