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NEAR-FIELD IMAGING OF AN INHOMOGENEOUS CAVITY WITH A MODIFIED FACTORIZATION METHOD*

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Abstract

This paper is concerned with the inverse problem of scattering of time-harmonic acoustic waves by an inhomogeneous cavity. We shall develop a modified factorization method to reconstruct the shape and location of the interior interface of the inhomogeneous cavity by means of many internal measurements of the near-field data. Numerical examples are carried out to illustrate the practicability of the inversion algorithm.

Mathematics subject classification: 35R30, 35Q60, 35P25, 78A46.
Key words: Inverse problems, Acoustic scattering, Numerical reconstruction, Inhomogeneous cavity.

1. Introduction

Consider the inverse scattering problem of recovering the shape and location of the interior interface of an impenetrable inhomogeneous cavity from many internal measurements. This kind of interior scattering problem may occur in many industry and military applications such as in material science, non-destructive testing and so on (cf. [14,32] and the references therein).

In contrast to the typical exterior scattering problem, where the incident field and the associated measurements are taken outside the objects, the interior scattering problem allows the incident point sources and measurements taken inside the cavity. In the current paper we consider the scattering of incident point sources by an inhomogeneous cavity, which is modeled by an inhomogeneous Helmholtz equation with various boundary conditions on the exterior boundary. Our goal is to study the inverse problem of numerically reconstructing the shape and location of the interior interface of the inhomogeneous cavity from the near-field data measured inside the cavity.

There are lots of investigations consisting of theoretical results and numerical methods for the exterior scattering problems in the past decades. We refer to the monographs [6,9,18] and the references quoted therein for detailed discussions. In the mean time the interior scattering problems also attracted many researchers' attention, where the studies are mainly focused on the numerical methods in recovering a homogeneous cavity. For example, the factorization method can be found in [19,21], the linear sampling method has been developed in [7,13,32,33] and a nonlinear integral equation method was established in [23]. It was also noticed that the

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inverse scattering by large cavities embedded in an infinite ground plane has been well studied, see, e.g. [2–4] and the references quoted therein. However, there are few results available in the literature for reconstructing an inhomogeneous cavity. For the case when the refractive index of the inhomogeneous cavity was described by a piecewise constant function, the authors in [24] obtained a uniqueness result on the identification of the interior interface as well as the piecewise constant refractive index, the technique can date back to the work [30], which focused on the shape reconstruction of the exterior inverse scattering problems.

In this paper, we intend to establish a modified factorization method as an analytical and numerical tool of reconstructing the interior interface of the inhomogeneous cavity with different kinds of boundary conditions on the exterior boundary. The factorization method was proposed by the work [15], which can provide a sufficient and necessary computational criterion for characterizing the shape and location of the scatterers. Therefore, it has been widely studied for various inverse scattering problems (cf. [5, 10, 11, 16–18, 25, 29] and the references therein). Recently, based on a technique of the detailed description of the kernel space of the related solution operator, the factorization method has been justified in [26] for the inverse problem of reconstructing the interior interface of a two-layered cavity in the case when the solution is discontinuous across the interior interface, that is, $u|_+ = u|_-$, $\partial_\nu u|_+ = \lambda\partial_\nu u|_-$ on ∂D for $\lambda \neq 1$. Unfortunately, the method developed in [26] is not capable of dealing with the inverse problem under consideration since the solution of our model problem is continuous across the interior interface, which means, $u|_+ = u|_-$, $\partial_\nu u|_+ = \lambda\partial_\nu u|_-$ on ∂D for $\lambda = 1$. This actually yields that the middle operator of the factorization in [26] is only compact for the problem setting we are considering. So, we need to develop a novel numerical algorithm, which is an approximate factorization method constructed depending on both the refractive index in the inhomogeneous medium and the boundary conditions on the exterior boundary, to solve our inverse problem. To be precise, we shall establish a series of perturbation operators N_m of the near field operator N, which is defined on a curve located inside the cavity. It can be proved that for sufficiently large $m_0 \in N_+$, the operator N_{m_0}, which can be viewed as a sufficiently small perturbation of the near field operator N, has a suitable factorization satisfying the Range Identity in [18, Theorem 2.15]. Moreover, $N_{m_0,\#}$ can also be viewed as a sufficient small perturbation of the noisy operator $N^\delta_\#$ with the noise level δ. Then the target interior interface of the inhomogeneous cavity can be approximately determined by means of the spectral data of $N_{m_0,\#}$ and $N^\delta_\#$. Numerical examples provided in Section 4 indeed illustrate the practicability of the proposed approximate factorization method.

The reader is also referred to [5, 15, 27, 28] for the justification of the factorization method for the exterior inverse medium scattering problems, to [1, 12, 31] for the iteration method, and to [8, 22] for the linear sampling method.

The paper is organized as follows. In Section 2, the formulations of the model problem will be presented. Some necessary properties on the solution operator are also provided. Section 3 is devoted to the mathematical establishment of an approximate factorization method for the inverse problem of reconstructing the interior interface of the inhomogeneous cavity. In Section 4, some numerical examples are carried out to illustrate the efficiency of the developed inversion algorithm.

2. Formulations of the Scattering Problem

We begin with the formulations of our scattering problem. Let $D \subset \mathbb{R}^d, d = 2, 3$, be an inhomogeneous cavity with a C^2 smooth boundary Σ. Assume further that $D = \overline{D_0} \cup D_1$