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A POSTERIORI ERROR ANALYSIS OF THE PML FINITE VOLUME METHOD FOR THE SCATTERING PROBLEM BY A PERIODIC CHIRAL STRUCTURE*

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Abstract

In this paper, we consider the electromagnetic wave scattering problem from a periodic chiral structure. The scattering problem is simplified to a two-dimensional problem, and is discretized by a finite volume method combined with the perfectly matched layer (PML) technique. A residual-type a posteriori error estimate of the PML finite volume method is analyzed and the upper and lower bounds on the error are established in the H^1-norm. The crucial part of the a posteriori error analysis is to derive the error representation formula and use a L^2-orthogonality property of the residual which plays a similar role as the Galerkin orthogonality. An adaptive PML finite volume method is proposed to solve the scattering problem. The PML parameters such as the thickness of the layer and the medium property are determined through sharp a posteriori error estimate. Finally, numerical experiments are presented to illustrate the efficiency of the proposed method.

Mathematics subject classification: 65N08, 65L60, 65N15, 35Q60.
Key words: Finite volume method, Perfectly matched layer, A posteriori error analysis, Chiral media.

1. Introduction

Consider a time-harmonic electromagnetic plane wave incident on a periodic chiral structure. The chiral structure is assumed to be periodic in x_1 direction and invariant in x_2 direction. The medium inside the structure is chiral, and two regions with homogeneous medium are separated by the periodic structure. From the point of view of mathematical modeling, our discussion on the scattering problem is simplified to the two-dimensional case. Recently, there has been still a considerable interest in the study of electromagnetic wave propagation by periodic chiral structure. In general, the electromagnetic wave propagation inside the chiral medium are governed by Maxwell equations together with the Drude-Born-Fedorov constitutive equations

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in which the electric and magnetic fields are coupled. The property of the chiral media is completely characterized by the chirality admittance β, the electric permittivity ε and the magnetic permittivity μ. On the other hand, periodic structures have generated great scientific interests in the past several years because of important applications in integrated optics, optical lenses, anti-reflective structures, lasers and so on.

Over the past two decades, scattering problem in chiral structures has gained a great development in the applied mathematical community. For the physical background and the model equations of the scattering problem inside chiral media, many literatures have discussed these issues and we refer to [1, 20, 23, 24, 34] on periodic and non-periodic chiral structures. From the computational aspects, lots of results and references on solving the chiral grating problem may be found in [1, 2, 35, 36]. For other related mathematical analysis and numerical methods of periodic achiral structures, the reader is referred to [3, 6, 15, 17, 18, 28] and references therein.

One of the difficulties for solving the scattering problem is to truncate the unbounded domain into a bounded computational domain with some adequate approximation accuracy. A popular and effective technique in truncating the unbounded domain is the perfectly matched layer (PML) method proposed by Berenger [8]. The key idea of the PML technique is to surround the computational domain by a special designed layer of finite thickness which can make the outgoing waves decay exponentially. At this point, a variety of PML methods have been developed and studied in the literature (cf. [25, 31]). Another difficulty for solving the scattering problem is to deal with the singularities of the solutions, an economical and effective method is the adaptive finite element method based on the a posteriori error estimate (cf. [5, 11, 13, 26, 27]). By using the PML technique in combination, the field of the adaptive finite element method attracted many researchers and has become more and more active in the numerical simulation of the scattering problem, we can refer to [7, 12, 14–16, 21, 22, 32] and references therein for the adaptive PML finite element methods and the related methods. The adaptive finite element methods combined with DtN or PML techniques are very attractive in solving the scattering problems, largely for this reason that DtN or PML method is applied to deal with the difficulty in truncating the unbounded domain and the adaptive finite element method can very efficiently capture the local singularities. However, to our best knowledge, there are very few works on the adaptive DtN or PML finite volume method for solving differential equations. For the literature, there are also some representative results on the posteriori error estimates and the adaptive computations of the finite volume method, the reader is referred to [9, 10, 19, 33] and references therein.

In this paper, we shall study the residual-type a posteriori error estimate of the PML finite volume method (PML-FVM) for solving 1D chiral grating problem. As the PML finite element method (PML-FEM) in [36], our PML finite volume method needs to surround the computational domain by a specially designed artificial layer which absorbs all waves coming from the computational domain. Meanwhile, compared with the DtN finite element method (DtN-FEM), our method can avoid dense blocks of the stiffness matrix generated by the computation of the discrete DtN operator. In this work, the a posteriori error estimate, which includes the finite volume discretization error and the PML error, is established by using similar arguments as the a posteriori error estimate of the PML-FEM. The main difficulty of our error analysis is that our PML-FVM is lack of the global Galerkin orthogonality in contrast to the DtN-FEM and PML-FEM. We overcome this difficulty by using an L^2-orthogonality property of the residual which plays a similar role as the Galerkin orthogonality. The error estimate is used to design the adaptive PML-FVM to choose elements for refinement and to determine the PML parame-