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A NON-MONOTONE SMOOTHING NEWTON ALGORITHM FOR SOLVING THE SYSTEM OF GENERALIZED ABSOLUTE VALUE EQUATIONS*

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Abstract

The system of generalized absolute value equations (GAVE) has attracted more and more attention in the optimization community. In this paper, by introducing a smoothing function, we develop a smoothing Newton algorithm with non-monotone line search to solve the GAVE. We show that the non-monotone algorithm is globally and locally quadratically convergent under a weaker assumption than those given in most existing algorithms for solving the GAVE. Numerical results are given to demonstrate the viability and efficiency of the approach.

Mathematics subject classification: 65F10, 65H10, 90C30.
Key words: Generalized absolute value equations, Smoothing function, Smoothing Newton algorithm, Non-monotone line search, Global and local quadratic convergence.

1. Introduction

The system of generalized absolute value equations (GAVE) is to find a vector $x \in \mathbb{R}^n$ such that

$$Ax + B|x| - b = 0, \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ and $0 \neq B \in \mathbb{R}^{n\times n}$ are two known matrices, $b \in \mathbb{R}^n$ is a known vector, and $|x|$ denotes the componentwise absolute value of $x \in \mathbb{R}^n$. In the literature, GAVE also occurs in the form of $Ax - B|x| = b$. In this paper, we do not make a distinction between it and (1.1) and

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put it down to GAVE (1.1). To the best of our knowledge, GAVE (1.1) was first introduced by Rohn in [34] and further investigated in [12,15,21,27,29,30] and references therein. Obviously, when $B = -I$ with I being the identity matrix, GAVE (1.1) becomes the system of absolute value equations (AVE)

$$Ax - |x| - b = 0, \tag{1.2}$$

which is the subject of numerous research works, see, e.g. [4,11,22,25,48,49] and references therein.

GAVE (1.1) and AVE (1.2) have attracted considerable attention in the field of optimization for almost twenty years, and the primary reason is that they are closely related to the linear complementarity problem (LCP) [25,30] and the horizontal LCP (HLCP) [27], which encompass many mathematical programming problems and have many practical applications [7,28]. In addition, GAVE (1.1) and AVE (1.2) are also bound up with the system of linear interval equations [33].

Due to the combinatorial character introduced by the absolute value operator, solving GAVE (1.1) is generally NP-hard [21, Proposition 2]. Moreover, if GAVE (1.1) is solvable, checking whether it has a unique solution or multiple solutions is NP-complete [30, Proposition 2.1]. Recently, GAVE (1.1) and AVE (1.2) have been extensively investigated in the literature, and the main research effort can be summarized to the following two aspects.

On the theoretical side, one of the main branches is to investigate conditions for existence, non-existence and uniqueness of solutions of GAVE (1.1) or AVE (1.2), see, e.g. [12,13,25,27, 30,34,43–45] and references therein. Specially, the following necessary and sufficient conditions that ensure the existence and uniqueness of solution of GAVE (1.1) can be found in [27,45] (see Section 2 for the definition of the column $\mathcal{W}$-property).

Theorem 1.1 ([27, Theorem 1]). *The following statements are equivalent:*

(i) *GAVE* (1.1) *has a unique solution for any* $b \in \mathbb{R}^n$.

(ii) $\{A + B, A - B\}$ *has the column* $\mathcal{W}$*-property.*

(iii) *For arbitrary nonnegative diagonal matrices* $D_1, D_2 \in \mathbb{R}^{n\times n}$ *with* $D_1 + D_2 > 0$,

$$\det\left[(A + B)D_1 + (A - B)D_2\right] \neq 0.$$

(iv) $A + B$ *is nonsingular and* $\{I, (A + B)^{-1}(A - B)\}$ *has the column* $\mathcal{W}$*-property.*

Theorem 1.2 ([45, Theorem 3.2]). *GAVE* (1.1) *has a unique solution for any* $b \in \mathbb{R}^n$ *if and only if matrix* $A + BD$ *is nonsingular for any diagonal matrix* $D = \operatorname{diag}(d_i)$ *with* $d_i \in [-1, 1]$.

It is easy to conclude that Theorems 1.1 and 1.2 imply that $\{A+B, A-B\}$ has the column $\mathcal{W}$-property if and only if matrix $A + BD$ is nonsingular for any diagonal matrix $D = \operatorname{diag}(d_i)$ with $d_i \in [-1, 1]$ (see Lemma 2.3 for more details). Throughout the theoretical analysis of this paper, we let the following assumption hold.

Assumption 1.1. *Let matrices A and B satisfy* $\{A + B, A - B\}$ *has the column* $\mathcal{W}$*-property.*

On the numerical side, there are various algorithms for solving AVE (1.2) or GAVE (1.1). For example, Mangasarian proposed concave minimization method [22], generalized Newton method [23], and successive linear programming method [24] for solving AVE (1.2). Zamani