Journal of Computational Mathematics
Vol.43, No.3, 2025, 540–568.

http://www.global-sci.org/jcm
doi:10.4208/jcm.2212-m2022-0122

PROXIMAL ADMM APPROACH FOR IMAGE RESTORATION WITH MIXED POISSON-GAUSSIAN NOISE*

Miao Chen
Department of Mathematics, Nanchang University, Nanchang 330031, China;
Department of Public Teaching, Quzhou College of Technology, Quzhou 324000, China
Yuchao Tang[1)]
School of Mathematics and Information Science, Guangzhou University, Guangzhou 510006, China
Email: yctang@gzhu.edu.cn
Jie Zhang
Department of Mathematics, The University of Hong Kong, Hong Kong SAR, China
Tieyong Zeng
Department of Mathematics, The Chinese University of Hong Kong, Hong Kong SAR, China

Abstract

Image restoration based on total variation has been widely studied owing to its edge-preservation properties. In this study, we consider the total variation infimal convolution (TV-IC) image restoration model for eliminating mixed Poisson-Gaussian noise. Based on the alternating direction method of multipliers (ADMM), we propose a complete splitting proximal bilinear constraint ADMM algorithm to solve the TV-IC model. We prove the convergence of the proposed algorithm under mild conditions. In contrast with other algorithms used for solving the TV-IC model, the proposed algorithm does not involve any inner iterations, and each subproblem has a closed-form solution. Finally, numerical experimental results demonstrate the efficiency and effectiveness of the proposed algorithm.

Mathematics subject classification: 65K10, 68U10, 94A08.
Key words: Image restoration, Mixed Poisson-Gaussian noise, Alternating direction method of multipliers, Total variation.

1. Introduction

Image restoration is a major problem in image processing, and its primary goal is to restore an original clean image from an observed image degraded by noise. Variational models are an important research direction for image restoration, wherein the basic idea is to construct an energy function according to a specific image restoration problem and minimize the energy function to obtain the original clear image. Generally, variational models include two items: a data fidelity item, which is established based on the probability distribution of noise, and a regularization item, which reflects prior information of the original image. Owing to the common influence of photon counting and thermal noise on the detector, observed images are often corrupted by mixed Poisson-Gaussian noise. Over the past two decades, researchers have extensively investigated the elimination of mixed Poisson-Gaussian noise. Consequently, the results of these studies have been applied to several practical problems, such as fluorescence microscopy images, X-ray computed tomography, and hyperspectral images. For more exam-

* Received May 8, 2022 / Revised version received August 29, 2022 / Accepted December 8, 2022 / Published online March 10, 2024 /
1) Corresponding author

ples, the interested readers are recommended to refer to [2, 6, 9, 12, 20–22, 39] and the references therein.

Compared with the problem of image restoration from single Gaussian noise or Poisson noise, the removal of mixed Poisson-Gaussian noise removal is more complex, owing to the need to establish a suitable data-fidelity term. Existing methods developed to address this issue can be divided into two categories. The first category involves a certain transformation in which the mixed Poisson-Gaussian noise is transformed into a single type of noise. Then, either a mainstream Gaussian denoising algorithm or Poisson denoising algorithm is used to denoise the image to obtain a clean image. Representative models include the generalized Anscombe transform model [18, 19, 25, 31], reweighted L^2 (WL^2) model [14, 29], and shifted Poisson model [10]. The advantage of such a type of model is that a large number of Gaussian denoising algorithms or Poisson denoising algorithms are available for selection. See, for example [1, 8, 30]. However, its biggest shortcoming is that the estimation of data fidelity in this type of model is not sufficiently accurate. The other category involves establishing a model directly based on the probability distribution of mixed Poisson-Gaussian noise. Based on the maximum a posteriori (MAP) estimation framework, Chouzenoux *et al.* [7] proposed an exact mixed Poisson-Gaussian model, in which a data fidelity term was established by combining the statistical characteristics of both Poisson and Gaussian noise. Additionally, they proved the convexity and gradient Lipschitz continuity of the data fidelity term. Simultaneously, the author employed this property to solve the model using the primal-dual splitting method. However, the exact mixed Poisson-Gaussian model needs to solve the infinite sum problem of the function term series; therefore, obtaining a numerically accurate solution is rather difficult. In contrast, the authors of [5, 17] proposed a total variation infimal convolution model by using the generalized joint MAP [17] estimation method. The TV-IC model has a simple formulation and low computational complexity. Additionally, it provides a good estimate of the mixed Poisson-Gaussian noise. Lanza *et al.* [17] proposed a primal-dual-based iterative algorithm to solve the TV-IC model. However, the algorithm requires Newton iterations to solve a nonlinear optimization subproblem, which significantly increases the amount of calculation required for the outer loop. Moreover, the convergence of the algorithm is not certain. Calatroni *et al.* [5] proposed a semi-smooth Newton algorithm to solve the TV-IC model. However, the algorithm is constrained by the use of Newton iterations to solve the subproblem, which renders it highly time-intensive. Zhang *et al.* [40] proposed a bilinear constraint-based ADMM (BCA) algorithm to solve the TV-IC model. However, the BCA algorithm requires inner iterations while solving subproblems. Besides, it is only suitable to denoise pure Poisson-Gaussian noise. Recently, Toader *et al.* [35] proposed a primal-dual hybrid gradient (PDHG) algorithm to solve the TV-IC model. However, this algorithm also involves a subproblem that must be solved using the Newton iteration method.

In addition to these two types of methods, data-driven deep learning models have also been applied to the mixed Poisson-Gaussian noise problem, and they have achieved good results. For example, Remez *et al.* [28] implemented a denoising model based on a class-aware strategy using a fully convolutional neural network. Although deep learning models exhibit superior performance compared to traditional variational models in some cases, they involve certain limitations in terms of network construction and model training. In particular, the generalization ability of deep learning models depends on the choice of the training dataset.

To overcome the complications encountered by existing algorithms in solving the TV-IC model, we propose a proximal bilinear constraint-based ADMM (PBCA) algorithm. The sig-