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A DRS-BASED PATH-FOLLOWING ALGORITHM FOR LINEAR PROGRAMMING*

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Abstract

In this paper, we present a novel Douglas-Rachford-splitting-based path following (DRS-PF) method that rapidly obtains the solution of linear programming (LP) with high accuracy. It originates from the fixed-point mapping associated with DRS on the log-barrier penalized LP. A path-following scheme is then proposed to simultaneously update the iterates and the penalty parameter for accelerating the overall procedure. Its global convergence towards an optimal solution to the original problem is established under mild assumptions. Numerical experiments show that DRS-PF outperforms the simplex and interior point methods implemented in the academic software (CLP, HiGHS, etc.) in terms of the geometric mean of the running time on a few typical benchmark data sets. In some cases, it is even reasonably competitive to the interior point method implemented in Gurobi, one of the most powerful software for LP.

Mathematics subject classification: 65K05, 90C05, 90C25.
Key words: Linear programming, Douglas-Rachford splitting, Second-order method, Path-following.

1. Introduction

Given $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n$, we consider the following general form of linear programming:

$$\begin{aligned} \min_x \ & c^\top x, \\ \text{s.t.} \ \ & Ax = b, \\ & x_i \geq 0, \quad \forall\, i > n_f, \end{aligned} \tag{P}$$

where $0 \leq n_f \leq n$ denotes the number of free variables. Without loss of generality, we assume that $m \leq n$ and the coefficient matrix A has full row rank.

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As one of the most widely used optimization tools, LP has thrived in the fields of finance, economics, management, engineering, transportation, scheduling, and many others [4, 29, 31]. The state-of-the-art methods for solving LP include the simplex method [7, 8] and the interior point methods (IPM, [16]). After decades of development, these methods can robustly obtain highly accurate solutions. When properly implemented, such as in commercial mathematical programming software (Gurobi [14], MOSEK [23], COPT [11], MindOpt [21], etc.), general LP problems can be solved efficiently.

However, both algorithms leaves room for further improvements in some scenarios, respectively. On the one hand, the simplex method visits one vertex of the feasible polytope for each iteration, causing too many iterations when tackling ill-conditioned LPs. On the other hand, although the IPM normally converges within a few iterations, its ability to take advantage of the information from the obtained solution to warm-start the solving process is limited, especially when facing a series of LPs whose data (i.e. A, b, c) are almost the same but each of which is perturbed by a small scale.

In recent years, a class of first-order methods (FOM) has been proposed to solve large-scale optimization problems, such as the primal-dual hybrid gradient (PDHG) method [5], the alternating directions method of multipliers (ADMM) [3], and the Douglas-Rachford splitting [10]. Note that the DRS can be expressed in the form of a fixed-point mapping [32]. The computational cost of one iteration of FOMs is much cheaper than that of the IPM. However, the well-known slow convergence of FOMs towards higher accuracy solutions causes them unsuitable for solving general LPs, where more accurate solutions are preferred. Recently, it is shown in [1] that the PDHG method can obtain solutions quickly on some very large-scale LPs when it is properly enhanced. But its performance on most of the instances in the Mittelmann data set [22] still lags behind the IPM.

We propose a new second-order method DRS-PF that works well in a few difficult scenarios. Our implementation of DRS-PF shows some desirable features that are not observed in the simplex method or the IPM, while at the same time achieves a superb speed in solving general LPs.

1.1. Our contributions

Our main contributions are listed as follows.

1. A novel construction of paths. Starting from the fixed-point mapping associated with the DRS, we transform the original LP and the log-barrier penalized LPs into a series of nonlinear equations (green and yellow parts in Fig. 1.1). Instead of solving them individually, we connect different equations by establishing a group of paths, along which the residual is proportional to the barrier parameter. The behavior of the paths when the barrier parameter goes to zero is meticulously analyzed, while the primal-dual optimal solutions of (P) can then be retrieved from the relationship established by DRS. This novel approach of constructing paths builds a link between different fixed-point mappings from the DRS. It can be possibly generalized to other FOMs and optimizations problems (e.g. quadratic programming, second-order cone programming, etc.).

2. An efficient path-following algorithm. Based on these paths, we design a path-following scheme DRS-PF, which employs a second-order method to update the iterate z and μ along the tangent direction of the path. After each update, a new path is constructed from the current iterate z and μ. This scheme (blue part in Fig. 1.1) allows us to update the iterate and