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A NEW SECOND ORDER NUMERICAL SCHEME FOR SOLVING DECOUPLED MEAN-FIELD FBSDES WITH JUMPS*

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Abstract

In this paper, we consider the numerical solution of decoupled mean-field forward backward stochastic differential equations with jumps (MFBSDEJs). By using finite difference approximations and the Gaussian quadrature rule, and the weak order 2.0 Itô-Taylor scheme to solve the forward mean-field SDEs with jumps, we propose a new second order scheme for MFBSDEJs. The proposed scheme allows an easy implementation. Some numerical experiments are carried out to demonstrate the stability, the effectiveness and the second order accuracy of the scheme.

Mathematics subject classification: 60H10, 60H35, 65C05, 65C30.
Key words: Mean-field forward backward stochastic differential equation with jumps, Finite difference approximation, Gaussian quadrature rule, Second order.

1. Introduction

To characterise the jumps in a Lévy process on a given probability space $(\Omega, \mathcal{F}, P)$, we introduce the Poisson random measure μ on $\mathsf{E} \times [0,T]$

$$\begin{aligned} \mu: \quad \Omega \times \mathcal{E} \times [0,T] \ &\rightarrow \ \mathbb{N}, \\ (\omega, A, [0,t]) \ &\rightarrow \ \mu(A \times [0,t]), \end{aligned}$$

where $\mathsf{E} = \mathbb{R}^q \backslash \{0\}$ and $\mathcal{E}$ is its Borel field. For given $t \in [0,T]$ and $A \in \mathcal{E}$, $\mu(A \times [0,t])$ is a random variable counting the number of jumps occurring in $[0,t]$ whose jump sizes belong to A. We usually suppress ω in μ for simplicity.

We call the measure $\nu: \mathsf{E} \times [0,T]$ defined by $\nu(A \times [0,t]) = \mathbb{E}[\mu(A \times [0,t])]$ the intensity measure of μ. Suppose that $\nu(de, dt) = \lambda(de)dt$ with λ being a Lévy measure on $(\mathsf{E}, \mathcal{E})$ satisfying $\int_{\mathsf{E}} (1 \wedge |e|^2) \lambda(de) < +\infty$, then the compensated Poisson random measure is defined as

$$\tilde{\mu}(de, dt) = (\mu - \nu)(de, dt) = \mu(de, dt) - \lambda(de)dt$$

such that $\{\tilde{\mu}(A \times [0,t])\}_{0 \leq t \leq T}$ is a martingale for any $A \in \mathcal{E}$ with $\lambda(A) < \infty$. Moreover, let F and ρ be the distribution and the probability density function of the jump size e, respectively, then it holds that

$$\lambda(de) = \lambda F(de) = \lambda \rho(e) de,$$

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where $\lambda = \lambda(\mathsf{E})$ is the intensity of μ. For more details of the Poisson random measure, the readers are referred to [6, 18].

Then we can get a complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ by letting $\mathbb{F} = \{\mathcal{F}_t\}_{0\le t\le T}$ be the filtration generated by the mutually independent m-dimensional Brownian motion W_t and the Poisson random measure μ, that is $\mathcal{F}_t = \mathcal{F}^0_{t+} \vee \mathcal{F}_0$, where $\mathcal{F}^0_{t+} = \bigcap_{s\ge t} \mathcal{F}^0_s$ with

$$\mathcal{F}^0_s = \sigma\big\{W_r,\ \mu(A\times[0,r])\,\big|\, A\in\mathcal{E},\ r\le s\big\},\quad s\in[0,T],$$

and the σ-field $\mathcal{F}_0 \subset \mathcal{F}$ satisfies:

- The Brownian motion W_t and the measure μ are independent of $\mathcal{F}_0$.
- $\mathcal{N}_p \subset \mathcal{F}_0$ with $\mathcal{N}_p$ being the set of all P-null subset of $\mathcal{F}$.

Now we consider decoupled mean-field forward backward stochastic differential equations with jumps on $(\Omega, \mathcal{F}, \mathbb{F}, P)$

$$\begin{aligned} X^{0,X_0}_t &= X_0 + \int_0^t \mathbb{E}\big[b\big(t, X^{0,x_0}_t, x\big)\big]\Big|_{x=X^{0,X_0}_s} ds + \int_0^t \mathbb{E}\big[\sigma\big(t, X^{0,x_0}_t, x\big)\big]\Big|_{x=X^{0,X_0}_s} dW_s \\ &\quad + \int_0^t\int_{\mathsf{E}} \mathbb{E}\big[c\big(s, X^{0,x_0}_{s-}, x, e\big)\big]\Big|_{x=X^{0,X_0}_{s-}} \tilde{\mu}(de, ds), \\ Y^{0,X_0}_t &= \mathbb{E}\big[\Phi\big(X^{0,x_0}_T, x\big)\big]\Big|_{x=X^{0,X_0}_T} + \int_t^T \mathbb{E}\big[f\big(s, \Theta^{0,x_0}_s, \theta\big)\big]\Big|_{\theta=\Theta^{0,X_0}_s} ds - \int_t^T Z^{0,X_0}_s dW_s \\ &\quad - \int_t^T\int_{\mathsf{E}} U^{0,X_0}_s(e)\tilde{\mu}(de, ds), \end{aligned} \tag{1.1}$$

where $t\in[0,T], x_0, X_0 \in \mathcal{F}_0$ is the initial values of mean-field forward stochastic differential equations with jumps (MSDEJs) and $\mathbb{E}[\Phi(X^{0,x_0}_T, x)]|_{x=X^{0,X_0}_T} \in \mathcal{F}_T$ with $\Phi: \Omega_d = \mathbb{R}^d\times\mathbb{R}^d \to \mathbb{R}^p$ is the terminal condition of mean-field backward stochastic differential equations with jumps (MBSDEJs); $b: [0,T]\times\Omega_d \to \mathbb{R}^d, \sigma: [0,T]\times\Omega_d \to \mathbb{R}^{d\times m}$, and $c: [0,T]\times\Omega_d\times\mathsf{E} \to \mathbb{R}^d$ are the drift, diffusion and jump coefficients of MSDEJs, respectively; $f: [0,T]\times\Omega_f \to \mathbb{R}^p$ is the so called generator of MBSDEJs with $\Omega_f = \mathbb{R}^d\times\mathbb{R}^p\times\mathbb{R}^{p\times m}\times\mathbb{R}^p\times\mathbb{R}^d\times\mathbb{R}^p\times\mathbb{R}^{p\times m}\times\mathbb{R}^p$; the term $\Theta^{0,x}_s = (X^{0,x}_s, Y^{0,x}_s, Z^{0,x}_s, \Gamma^{0,x}_s)$ with $x = x_0$ or X_0, and

$$\Gamma^{0,x}_s = \int_{\mathsf{E}} U^{0,x}_s(e)\eta(e)\lambda(de)$$

for some Borel function $\eta: \mathsf{E}\to\mathbb{R}$ satisfying $\sup_{e\in\mathsf{E}}|\eta(e)| < \infty$. We call a quadruplet $(X^{0,X_0}_t, Y^{0,X_0}_t, Z^{0,X_0}_t, U^{0,X_0}_t)$ an L^2-adapted solution of (1.1) if it is $\mathcal{F}_t$-adapted, square integrable and satisfies (1.1). In general, initial values x_0 and X_0 are different, and

$$\big(X^{0,x_0}_t, Y^{0,x_0}_t, Z^{0,x_0}_t, U^{0,x_0}_t\big) = \big(X^{0,X_0}_t, Y^{0,X_0}_t, Z^{0,X_0}_t, U^{0,X_0}_t\big)\big|_{X_0=x_0}.$$

In this paper, we shall numerically solve the solutions $(X^{0,X_0}_t, Y^{0,X_0}_t, Z^{0,X_0}_t, \Gamma^{0,X_0}_t)$ instead of $(X^{0,X_0}_t, Y^{0,X_0}_t, Z^{0,X_0}_t, U^{0,X_0}_t)$. Here the MFBSDEJs (1.1) is called decoupled because the coefficients of MSDEJs do not depend on the solutions of MBSDEJs.

In 2009, Buckdahn *et al.* [4] first studied the existence and uniqueness of the solutions of mean-field forward backward stochastic differential equations (MFBSDEs) in a general Markovian setting. Then based on those researches, Li [12] further proved the existence and unique-