Ann. Appl. Math.
doi: 10.4208/aam.OA-2024-0025

Vol. **41**, No. 2, pp. 239-266
May 2025

A Survey on the Performance of Krylov Subspace Methods in High Order Compact Schemes for Solving Poisson's Equation for Application in Incompressible Fluid Flow Solvers

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Received 3 December 2024; Accepted (in revised version) 22 May 2025

Abstract. The efficiency of three Krylov subspace methods with their ILU0-preconditioned version in solving the systems with the nondiagonal sparse matrix is examined. The systems have arisen from the discretization of Poisson's equation using the 4th and 6th-order compact schemes. Four matrix-vector multiplication techniques based on four sparse matrix storage schemes are considered in the algorithm of the Krylov subspace methods and their effects are explored. The convergence history, error reduction, iteration-resolution relation and CPU-time are addressed. The efficacy of various methods is evaluated against a benchmark scenario in which the conventional second-order central difference scheme is employed to discretize Poisson's equation. The Krylov subspace methods, paired with four distinct matrix-vector multiplication strategies across three discretization approaches, are tested and implemented within an incompressible fluid flow solver to solve the elliptic segment of the equations. The resulting solution process CPU-time surface gives a new vision regarding speeding up a CFD code with proper selection of discretization stencil and matrix-vector multiplication technique.

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AMS: 65F10, 76D05

Key words: High order compact, Krylov subspace methods, Navier-Stokes equations, Poisson's equation, CPU-time, matrix-vector multiplication, sparse storage schemes.

1 Introduction

In the last three decades, many attempts have been made to develop high-order compact (HOC) difference schemes for solving Navier-Stokes equations [1,2]. Wang et al. [3] developed a fourth-order compact difference scheme for the steady stream function-vorticity formulation of the 2D incompressible Navier-Stokes equations on non-uniform grids. The Maple software package was employed for this purpose. To address boundary layers, grid transformation techniques were applied, facilitating the mapping of a non-uniform grid to a uniform one suitable for the fourth-order compact difference scheme. Additionally, they utilized a Krylov subspace method along with an ILUT preconditioning technique to effectively solve the ensuing linear system. Li et al. [4] also proposed a compact fourth-order finite difference scheme for steady incompressible Navier-Stokes equations, focusing on achieving high accuracy for steady-state solutions with structured grids. Their work emphasizes the efficiency of compact schemes in handling incompressible flow problems, which complements the developments discussed in this study. Pandit et al. [5] proposed an implicit high-order compact (HOC) finite-difference scheme for solving the two-dimensional unsteady Navier–Stokes equations on irregular geometries on orthogonal grids. In addition to incorporating the favorable aspects of HOC schemes, their formulation possesses the added benefit of effectively representing transient viscous flows that encompass free and wall-bounded shear layers. These flows inherently exhibit variations in spatial scales. Tian and Dai [6] proposed a class of high-order compact (HOC) exponential finite difference (FD) methods for solving one- and two-dimensional steady-state convection-diffusion problems. They demonstrated the non-oscillatory property and high accuracy approximation solution of their scheme as well as its suitability for convection-dominated problems. For the application of HOC finite difference scheme for solving the unsteady two-dimensional (2-D) convection-diffusion equations, one can also refer to Kalita et al. [7] and Kalita, Chhabra [8]. Boersma [9] developed a staggered compact high order (up to 12th-order) numerical method to solve the compressible Navier-Stokes equations and extended his work in the case of incompressible Navier-Stokes equations [10]. Ray [11] used a higher-order compact (HOC) finite difference scheme for capturing the very complex flow phenomenon of unsteady flow past a rotating and translating circular cylinder. He considered the stream function-vorticity