

Preface

Special Issue Dedicated to Professor Avy Soffer's 70th Birthday (Part I)



Professor Avraham (Avy) Soffer was born in Israel on December 28, 1955. He earned his Bachelor's, Master's, and Doctor's degrees in Science from Tel-Aviv University in 1976, 1980, and 1984, respectively. Between 1978 and 1983, he worked as an assistant at Tel-Aviv University.

In 1981, he was a visiting scholar at the University of Bourgogne, France. He completed a postdoctoral fellowship at the Weizmann Institute in 1984, followed by a Weizmann Fellowship at the California Institute of Technology from 1985 to 1986. In 1987, he was a Bantrell Fellow at the California Institute of Technology. He joined Princeton University as an Assistant Professor from 1987 to 1993, before moving to Rutgers University, where he served as an Associate Professor from 1993 to 2000. In 2000, he was promoted to full Professor at Rutgers University. From 2003 to 2004, he also held a position at the Institute for Advanced Study, Princeton. In 2006, he was honored with the title of Distinguished Professor at Rutgers University.

Throughout his career, Professor Soffer has held various visiting positions, including at the University of Cergy-Pontoise, France (2004–2006), the Technion, Israel (2007, 2012), and the Weizmann Institute, where he was appointed Weston Professor in 2008. He was also a visiting professor at the Institut des Hautes Études Scientifiques (IHES), France

(2008, 2009), the University of Paris-Sud, Orsay (2008), and the École Polytechnique, France (2009). Since 2013, he has been a member of the Physics Graduate Faculty at Rutgers University.

Professor Soffer's awards and prizes mainly include the Avraham Cohen Fellowship in Physics in 1982, a plenary talk at the International Conference of Mathematical Physics in Marseille in 1986, the Alfred P. Sloan Fellowship in Mathematics (1988–1989), and the Rutgers Research Council Grant Awards in 1996 and 2006. In 2006, he delivered an invited 45-minute lecture at the International Congress of Mathematicians (ICM) in Madrid, Spain. He was awarded the Weston Fellowship at the Weizmann Institute (2007–2008) and was named a Fellow of the American Mathematical Society (AMS) in 2015.

He is a member of DIMACS (the Center for Discrete Mathematics and Theoretical Computer Science) and serves on the editorial board of several book series. From 2001 to 2013, he was on the editorial board of Geometric and Functional Analysis (GAFA), and since 1999, he has been an associate editor for Letters in Mathematical Physics.

Professor Soffer has made significant contributions in the field of mathematical physics and partial differential equations. His research topics cover quantum and tunneling dynamics and scattering, spectral theory and resonances, dispersive wave equations, linear and nonlinear, soliton dynamics, and scattering on manifolds. His works led to new predictions later verified by experiments. Some of his main scientific achievements can be summarized as follows:

1. Time Dependent Resonance Theory. Avy Soffer and his collaborators [2] presented a time-dependent method as a tool to solve a class of resonance problems, which is about perturbations of self-adjoint operators having embedded eigenvalues in their continuous spectra. The main assumptions are (i) the emergence of the nonlinear Fermi golden rule; (ii) the local decay estimates for self-adjoint operators. The approach can yield information about thresholds and doesn't require the potential to be dilation analytic. It can be used to present the asymptotic behavior of solutions of an infinite dimensional Hamiltonian system and is also adaptable to nonautonomous linear, and nonlinear problems. Besides, they proved that an embedded eigenvalue is unstable at a threshold energy under suitable hypotheses. Their work is conducive to the study of many physical systems, e.g., the coupling of an atom or molecule to a photon-radiation field, and Auger states of the helium atom, as well as in spectral geometry and number theory.

2. Nonlinear Waves. Avy Soffer and his collaborator M. I. Weinstein [1] studied a class of nonlinear Klein-Gordon equations which are Hamiltonian and are perturbations of linear dispersive equations. In the unperturbed linear system, there exists a bound state, which is a spatially localized and time-periodic solution. When the linear system is perturbed by generic nonlinear Hamiltonian perturbations, they showed that spatially localized and time-periodic solutions of the linear problem are destroyed via slow radiation of energy to infinity. However, the destruction is an anomalously slow process, therefore these solutions are observed as "metastable states". This work introduces several new ideas and techniques, such as the time-dependent method for the quantum resonance problem and perturbations of embedded eigenvalues. Notably, it lays the groundwork for developing a formalism to

compute normal forms for finite-dimensional Hamiltonian systems coupled to a radiation field in which the radiation introduces a damping effect to the normal form. In addition, this work has inspired a large number of related research in this area. See also the relevant work in [3] for the nonlinear Schrödinger equations.

3. Quantum Theory and Scattering. Professor Avy Soffer and his collaborator I. M. Sigal [6, 7] showed the asymptotic completeness for short-range quantum-mechanical systems consisting of an arbitrary number of particles. Asymptotic completeness in the quantum mechanics of N -particle systems refers to a situation where the long-term behavior of the system is fully understood in terms of its scattering processes. Specifically, it states that every state of the N -particle system that is not bound (i.e., not a discrete bound state or eigenfunction of the system's Hamiltonian) will eventually behave like a system of independent clusters of particles which moves independently from each other as time tends to infinity. Other related results can also be found in his papers [4, 5, 8, 9].

4. Decay Estimates for Schrödinger Operators. This is another main topic in Professor Avy Soffer's research, see for example [10] which investigates the decay estimates of Schrödinger operator $H = -\Delta + V(x)$. This topic has been active topic of research in the last thirty years. Professor Avy Soffer (jointed with Journ and Sogge) firstly established the L^p -time decay estimates in regular case when $n \geq 3$. This decay estimate applied broadly to nonlinear Schrödinger equations. The main one is to extend the "global decay" estimate of Strichartz. Also, the estimate extends the "local decay" estimates for $V = 0$ and the weighted estimates of A. Jensen and T. Kato [Duke Math. J., 46(3) (1979), 583–611], J. B. Rauch [Commun. Math. Phys., 61(2) (1978), 149–168] and Jensen [Duke Math. J., 47(1) (1980), 57–80].

5. Scattering and Dispersion on Manifolds. Together with his former student P. Blue [13–15], he developed Morawetz-type a-priori estimates for wave equations on Schwarzschild Manifolds. This has led to a large number of follow-up works in general relativity related problems, in particular black-hole Dynamics.

6. Random Fields Ising Models. In joint works with M. Schwartz [11, 12] on the problem of critical phenomena of Random Fields Ising Models (RFIM), he derived correlation inequalities, known as Schwartz-Soffer Inequalities, which also led to bounds and sharp estimates on critical exponents.

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