

Non-Global Existence of Regular Solution to Initial Value Problem of Relativistic Euler Equations in $\mathbb{R}^N$

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Received 30 April 2024; Accepted (in revised version) 2 June 2024

Dedicated to the celebration of the 70th birthday of Professor Avy Soffer

Abstract. In fluid mechanics and astrophysics, relativistic Euler equations can be used to describe the effects of special relativity which are an extension of the classical Euler equations. In this paper, we will consider the initial value problem of relativistic Euler equations in an initial bounded region of $\mathbb{R}^N$. If the initial velocity satisfies

$$\max_{\vec{x}_0 \in \partial\Omega(0)} \sum_{i=1}^N v_i^2(0, \vec{x}_0) < \frac{c^2 A_1}{2},$$

where A_1 is a positive constant depend on some sufficiently large T^* , then we can get the non-global existence of the regular solution for the N -dimensional relativistic Euler equations.

AMS subject classifications: 35A01, 35E15 , 35Q75

Key words: Non-global existence, relativistic Euler equations, regular solution, initial value problem.

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1 Introduction and main result

It is well-known that when macroscopic speed of the fluid is close to the speed of light, or when the microscopic speed of the fluid particles is large, the motion of the fluid no longer obeys the Newtonian laws [1]. Thus, we need to consider the relativistic effect of the fluids. The relativistic Euler equations are used to explore the effects of special relativity in fluid mechanics and astrophysics [2]. The isentropic Euler equations in relativistic fluid dynamics can be written as following

$$\begin{cases} \partial_t \left(\frac{n}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \right) + \nabla \cdot \left(\frac{n\vec{v}}{\sqrt{1-\frac{|\vec{v}|^2}{c^2}}} \right) = 0, \\ \partial_t \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} - \frac{p}{c^2} \right) + \nabla \cdot \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \right) = 0, \\ \partial_t \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \right) + \nabla \cdot \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \otimes \vec{v} \right) + \nabla p = 0, \end{cases} \quad (1.1)$$

where $n, \rho = \rho(t, \vec{x}) \geq 0$ and $\vec{v} = v(t, \vec{x}) = (v_1, v_2, \dots, v_N) \in \mathbb{R}^N$ denote the charge density, the mass-energy density, the velocity of an electro-fluid and $\vec{x} = (x_1, x_2, \dots, x_N) \in \mathbb{R}^N$, respectively. Here c is the speed of light and p is the pressure, where $\gamma > 1$ is the adiabatic index. In addition, n and ρ satisfy $\rho = n(1 + \frac{e}{c^2})$, where $e \geq 0$ is the special internal energy. According to the first thermodynamics law [1], we have

$$\theta dS = de + pd \left(\frac{1}{n} \right) = \frac{d\rho}{n} - \frac{p + \rho c^2}{n^2 c^2} dn, \quad (1.2)$$

where the θ is the temperature and S is the entropy. In the isentropic case, we get

$$\frac{dn}{nc^2} = \frac{d\rho}{p + \rho c^2}. \quad (1.3)$$

If the pressure p depends only on the mass-energy density ρ , which satisfies $p = \rho^\gamma$, and the system of energy and momentum conservation laws is closed, system (1.1) can be reduced to the following system (see [3])

$$\begin{cases} \partial_t \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} - \frac{p}{c^2} \right) + \nabla \cdot \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \right) = 0, \\ \partial_t \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \right) + \nabla \cdot \left(\frac{\rho c^2 + p}{c^2 - |\vec{v}|^2} \vec{v} \otimes \vec{v} \right) + \nabla p = 0. \end{cases} \quad (1.4)$$

In the following, we consider the initial value problem of equations (1.4) with initial data

$$t=0, \quad \rho = \rho_0(\vec{x}), \quad \vec{v} = \vec{v}_0(\vec{x}). \quad (1.5)$$