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The Characteristics of Solutions to Semilinear Wave Equation with Logarithmic Plus Polynomial Nonlinearities

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Dedicated to the celebration of the 70th birthday of Professor Avy Soffer

Abstract. The semilinear wave equation with logarithmic and polynomial nonlinearities is considered in this paper. By adjusting and using potential well method, we attain the global-in-time existence and infinite time blowup solutions at subcritical initial energy level $E(0)<d$. Then using additional conditions on initial data, these results are enlarged at critical case $E(0)=d$ and arbitrarily positive case $E(0)>0$.

AMS subject classifications: 35L05

Key words: Global existence, blowup, logarithmic and polynomial combined nonlinearity, potential well.

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1 Introduction

In this article, we are exploring the initial boundary value problem of the semilinear wave equation containing logarithmic plus polynomial source terms

$$\begin{cases} u_{tt}-\Delta u=u\ln|u|^k+|u|^{p-1}u, & x\in\Omega, \quad t>0, \\ u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), & x\in\Omega, \\ u(x,t)=0, & x\in\partial\Omega, \quad t>0, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^n$ is a smooth bounded domain and $k>1$, $2<p<\frac{n+2}{n-2}$ $(n\geq 3)$ are constants. The semilinear hyperbolic equations are very important nonlinear evolution equations in the field of mathematical physics. The polynomial nonlinearities model the external force that enhances energy and drives the system toward possible instability [12]. The evolution equations with logarithmic nonlinearities come naturally in inflation cosmology and supersymmetric field theory (see [4, 18]). Moreover, we can see its implementation in different area of physics for example nuclear physics, optics, and geophysics (see [5, 16]). According to existing literature, some special analytical solutions of the problem with logarithmic nonlinearities can be found in the logarithmic quantum mechanics (see [3, 25]).

In order to recall the related work of problem (1.1), we give the following hyperbolic model with general nonlinearity $f(u)$ to discuss different cases of the nonlinearity

$$u_{tt}-\Delta u=f(u). \tag{1.2}$$

The background survey will be started with very important work of Sattinger [29], which was revolutionary for investigating nonlinear wave equations. The author [29] first introduced the concept of potential well W to study the above semi-linear wave equation with polynomial source term when initial data u_0 lie in the so-called potential well to get the solution still belongs to the potential well as described above. For precisely specified class of initial data finite blowup result was also studied. In [27], Payane and Sattinger proved finite blowup of solution of the problem (1.2) while u_0 lies outside the potential well W. Besides, they discussed potential energy J with availability of saddle point and gave explanatory description of potential well W. The way to search blow up results for the abstract problem (1.2) was first developed in [21]. In [2] point-wise blow up in finite time was shown for (1.2). Using assumption $(u_0,u_1)\geq 0$, the proof of blow up (global nonexistence) was treated in [17] and [28] for definitely positive initial energy case. The so-called family of potential wells was proposed by Liu Yacheng [20] which incorporates single potential well W as a special case, and the previous results were developed in $E(0)<d$ for special