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Existence of Global Attractor for Weakly Damped FDS Nonlinear Wave Equations

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Abstract. The paper investigates the well-posedness of global solutions and the existence of global attractors for weakly damped FDS nonlinear wave equations. It establishes the well-posedness of weak solutions using Galerkin approximation and a priori estimate. Subsequently, a dynamical system is constructed based on the well-posedness of the solution. The existence of a bounded absorbing set for the equations and the smooth properties of the operator semigroup are presented, leading to the existence of a global attractor.

AMS subject classifications: 35B40, 35B41, 35B45

Key words: Global attractors, a priori estimate, FDS nonlinear wave equations.

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1 Introduction

This paper mainly considers weakly damped FDS nonlinear wave equations in a bounded domain. The equations are given by

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$$\begin{cases} u_{tt}-\Delta u+\alpha^2 v^2 u+\gamma_1 u_t+\kappa_1 u=f(x), \\ v_{tt}-\Delta v+\alpha^2 v|u|^2+\dfrac{1}{2}(v^3-v)+\gamma_2 v_t+\kappa_2 v=0, \\ u(x,0)=u_0, \quad u_t(x,0)=u_1, \\ v(x,0)=v_0, \quad v_t(x,0)=v_1, \\ u|_{x\in\partial\Omega}=0, \quad v|_{x\in\Omega}=0, \end{cases} \tag{1.1}$$

where $u(x,t)$ is a complex-valued function, $v(x,t)$ is a real-valued function, dissipative coefficients $\alpha,\kappa_1,\gamma_1,\gamma_2>0$, $\kappa_2>\frac{1}{2}$, and $f(x)\in L^2(\Omega)$. When the damping coefficients $\kappa_1,\kappa_2,\gamma_1,\gamma_2=0$, Eqs. (1.1) are called FDS nonlinear wave equations. In 1976, Friedberg-Lee-Sirlin [6] proposed a class of three-dimensional solitary wave problems involving scalar fields. They outlined the essential conditions for non-topological solitary wave solutions and presented a general theory of stability based on quantum mechanics, along with numerical findings for specific cases. Previous literature highlighted that Eqs. (1.1) studied by Friedberg-Lee-Sirlin [7] fall under the category of FDS nonlinear wave equations. Subsequently, Zhou et.al [17] generalized the problem to cases with non-Abel internal symmetry. Guo [16] used the method of abstract differential operators to establish the existence and uniqueness of global solutions for three-dimensional FDS nonlinear wave equations.

The study of the asymptotic behavior of dynamical systems is one of the most important problems in contemporary mathematical physics. For dissipative autonomous systems derived from evolution equations in mathematical physics, the long-term behavior of solutions can be described using global attractors. Research on the existence of global attractors is an important approach to analyzing and solving problems in dissipative dynamical systems.

In the realm of a single wave equation, extensive research has been conducted [4, 5, 8]. These studies can be categorized based on the type of damping term they incorporate, such as weak damping, strong damping, and structural damping. For instance, consider the nonlinear wave equation with a damping term given by

$$u_{tt}-\Delta u+\gamma(-\Delta)^a u_t+f(u)=g(x), \tag{1.2}$$

where the damping coefficient $\gamma>0$, $f(u)$ is the nonlinear term, and $g(x)$ is the external forcing term. When $a=0$, u_t is referred to as a weak damping term. Pata-Zelik [10] studied wave equations with weak damping and proved the existence of a global attractor for such equations. Ball [2] previously established the existence of a global attractor for wave equations with weak damping without external forcing terms. When $a=1$, $-\Delta u_t$ is called a strong damping term, and Pata-Squassina [9] obtained the existence of a global attractor for strongly damped wave equations. In