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Oscillation-Eliminating Discontinuous Galerkin Methods for Multicomponent Chemically Reacting Flows

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Abstract. This paper proposes a robust and efficient oscillation-eliminating discontinuous Galerkin (OEDG) method for solving multicomponent chemically reacting flows, which is an extension and application of the recent work [M. Peng, Z. Sun, and K. Wu, Math. Comput., 2024, doi.org/10.1090/mcom/3998]. Following recently developed high-order bound-preserving discontinuous Galerkin method in [J. Du and Y. Yang, J. Comput. Phys., 469 (2022), 111548], we incorporate an OE procedure after each Runge-–Kutta time stage to suppress spurious oscillations. The OE procedure is defined by the solution operator of a damping equation, which can be analytically solved without requiring discretization, making its implementation straightforward, non-intrusive, and efficient. Through careful design of the damping coefficients, the proposed OEDG method not only achieves the essentially non-oscillatory (ENO) property without compromising accuracy but also preserves the conservative property—an indispensable aspect of the bound-preserving technique introduced in [J. Du and Y. Yang, J. Comput. Phys., 469 (2022), 111548]. The effectiveness and robustness of the OEDG method are demonstrated through a series of one- and two-dimensional numerical tests on the compressible Euler and Navier–Stokes

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equations for chemically reacting flows. These results highlight the method's capability to handle complex flow dynamics while maintaining stability and high-order accuracy.

AMS subject classifications: 65M12, 65M60, 76N06

Key words: Oscillation elimination discontinuous Galerkin (OEDG) method, modal filter, bound-preserving, multicomponent chemically reacting flows.

1 Introduction

Multicomponent chemically reacting flows play a vital role in modeling and simulating various complex phenomena, such as inertial confinement fusion, high-speed combustion, detonation waves, and cavitation bubbles. These flows involve intricate interactions between multiple chemical species, often under extreme conditions, posing significant challenges for numerical modeling. A key difficulty is accurately capturing sharp features like shock waves and detonation fronts while ensuring numerical stability and precision. Uncontrolled oscillations can lead to nonphysical results, such as negative values for density or pressure, ultimately causing simulations to fail. In detonation scenarios, the rapid reaction rates introduce stiff source terms, which impose severe time-step restrictions when using explicit time integration methods.

While high-order numerical methods have achieved considerable success in single-component compressible flows, they often encounter difficulties in chemically reacting flows where the specific heat ratio varies with composition and temperature, leading to nonphysical oscillations and inaccuracies. Various approaches, such as low-dissipation methods [21], the ghost fluid method [22], entropy-stable methods [34], and interface-capturing techniques [24], have been proposed to address these challenges. Additionally, adaptive methods [3, 35] and moving mesh strategies [36] aim to enhance computational efficiency. However, these techniques often struggle to achieve a consistent balance between minimizing dissipation and preserving physical bounds, particularly in the context of high-order methods. This paper focuses on discontinuous Galerkin (DG) methods for multicomponent chemically reacting flows [2,13,16,24,25,32,33]. Initially introduced by Reed and Hill [40] for neutron transport problems, DG methods were later generalized by Cockburn and Shu to solve time-dependent partial differential equations, including hyperbolic conservation laws and convection-diffusion equations [6–10]. Since then, DG methods have been extensively developed and applied in computational fluid dynamics