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Comparison of Sharp and Diffuse Interface Methods for Radially Symmetric Compressible Multi-Medium Fluid Flows

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Abstract. The effects of non-physical mixing on interface development are still not reasonably evaluated when diffuse interface methods (DIMs) are employed to treat the two-medium flows with immiscible interfaces, especially for compressible multi-medium flows with geometrical source terms. In this work, we simulate radially symmetric multi-medium flows employing the sharp interface methods (SIMs) and DIMs to evaluate their performance such as pressure dislocations, mass conservation, and convergence. The γ-based model and the five-equation transport model are two common DIMs, which are extended to equations with geometrical source terms combined with discontinuous Galerkin (DG) methods. For the SIMs, we employ the classical modified ghost fluid method (MGFM) and its second-order extension (2nd-MGFM) developed recently. Numerical results exhibit that the 2nd-MGFM is more effective in maintaining the interfacial pressure equilibrium relative to the MGFM. The DIMs can always maintain the pressure continuity naturally and total mass conservation, which is not available for SIMs. Further, under the premise of immiscible interfaces defined artificially, the DIMs cannot provide satisfactory single medium mass conservation, while the SIMs have a smaller mass error. In addition, compared to the other three methods, the 2nd-MGFM has higher confidence for radially symmetric flows, matching the exact solution in sparser grids.

AMS subject classifications: 65M22, 65M60

Key words: Multi-medium compressible flow, sharp interface methods, diffuse interface methods, modified ghost fluid method, γ-based model, five-equation transport model.

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1 Introduction

The numerical simulation of multi-medium flows has important applications in many fields [1–9]. The major difficulty in the numerical simulation of multi-medium flows lies in the proper treatment of the material interface and its vicinity. Under the Eulerian framework, the numerical schemes can be generally classified into two major categories: sharp interface methods (SIMs) [8, 10–16] and diffuse interface methods (DIMs) [4, 9, 17–27].

The diffuse interface methods treat the interface as a diffused layer with finite thickness and permit it to diffuse numerically within the region. Various different models [6,18,28–33] were developed with the introduction of additional equations to describe characteristic variables, such as volume fraction, mass fraction, or function of material parameters. This leads to potential difficulties in numerical discretization since thoses models are usually described by a non-conservative system [34]. It is not easy to correctly resolve the thermodynamic state while remaining unaffected by numerical oscillations caused from numerical mixing around the material interface. In addition, the diffuse interface methods confront the challenge of continuous spread for the mixed interface thickness. To control or mitigate the numerically deterioration of the interface diffusion, improving the algorithm accuracy and the inverse numerical dissipation technique are the fundamental tools, which impose high requirements on the stability of the algorithm. Correspondingly, the effect of non-physical mixing on the development of the interface is still not reasonably evaluated when used to treat the two-medium flows with immiscible interfaces. Specifically, it is not known whether the numerical results converge with grid refinement for the compressible multi-medium flows with geometrical source terms using the diffuse interface methods. Recently, He [35] have found that a nonphysical solution occurs near the interface due to nonconservative discretization at the interface, which leads to inaccurate numerical simulations. In addition, non-physical mixing in extreme environments provides some challenges for numerical simulations that require high fidelity. For example, Clark [36] pointed out that to evaluate ignition performance in inertial confinement fusion (ICF), it is necessary to achieve high-resolution, high-fidelity simulations of the interfaces evolution, such as the evolution of defective structures in target pellets.

In contrast, sharp interface methods [8, 10–16, 37–41] treats flow immiscible with a sharp material interface to separate different fluids distinctly. In such treatment, two major concerns have to be addressed. One is to capture the interface position, which can be tracked with a Lagrangian method [42, 43] or captured with the level set technique [44] or the volume of fluid method [45]. The other is to control the conservation error and correctly decompose wave interaction occurring possibly at the interface. A typical representative is the modified ghost fluid method (MGFM) [16] based on the multi-medium Riemann problem, and its variants [37, 46, 47] and so on, which have been successfully applied to solve various problems involving strong shocks interacting with gas-gas, gas-water and gas-water-solid interfaces [13, 16, 37, 39, 46–57] owing its effectiveness and ro-