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Two-Relaxation-Time Regularized Lattice Boltzmann Model for Navier-Stokes Equations

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Abstract. In this paper, we develop a two-relaxation-time regularized lattice Boltzmann (TRT-RLB) model for simulating weakly compressible isothermal flows, which demonstrates superior stability and accuracy over existing models such as the regularized lattice Boltzmann (RLB) and two-relaxation-time (TRT) models. In this model, a free relaxation parameter, $\tau_{s,2}$, is employed to relax the regularized non-equilibrium third-order terms. Chapman-Enskog analysis reveals that our model can accurately recover the Navier-Stokes equations. Theoretical analysis and numerical experiments both confirm the model's ability to eliminate non-physical numerical slip associated with the half-way bounce-back scheme. Our simulations of the double shear layer problem and Taylor-Green vortex flow exhibit pronounced advantages in terms of stability and accuracy, even under super-high Reynolds numbers as high as $\mathrm{Re}=10^7$. Additionally, the simulation of creeping flow around a square cylinder showcases the model's precision in computing ultra-low Reynolds numbers down to $\mathrm{Re}=10^{-7}$. This robust capability confirms the proposed model as a highly effective and adaptable tool in computational fluid dynamics.

AMS subject classifications: 76M25, 65M99

Key words: Regularized lattice Boltzmann, two-relaxation-time, high-Reynolds number, creeping flow, numerical slip.

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1 Introduction

In recent years, the lattice Boltzmann method (LBM) has gained significant attention in the realm of computational fluid dynamics (CFD) and related fields. Due to its unique features such as inherent parallelism, ease of handling complex boundary conditions, physical intuitiveness, and scalability, the LBM is increasingly applied in solving nonlinear partial differential equations [1–3] and simulating complex fluid problems, including turbulent flow [4–7], combustion [8–10], multiphase interactions [11–14], evaporation and phase change [11, 15–17], fluid-structure interaction [18, 19], porous media flow [20–22], and chemical reactions [23, 24], among others. Several books [25, 26] and review articles [11, 27] are recommended for readers seeking a deeper foundational understanding. However, many challenging issues in the lattice Boltzmann method arise from the nonlinear collision term, such as numerical instability, lack of Galilean invariance in transport coefficients, and the fixed and unadjustable Prandtl number. Adopting adaptive multi-resolution or more complex execution schemes, such as those in the series of works by Thomas Bellotti [28,29], is undoubtedly a promising approach. However, if we consider only the use of uniform standard lattices and employ the simplest collision and streaming schemes, improving the implementation of the collision process may be a universally recognized effective method. The standard approach for handling nonlinear collision terms is to employ the Bhatnagar-Gross-Krook (BGK) approximation [30], known as the lattice BGK (LBGK) model, which was developed by Qian et al. [31]. In this model, the probability particle distribution functions relax towards equilibrium at a single rate, a simplicity that has contributed to its widespread popularity. While the LBGK model suffices for problems with moderate Reynolds numbers, it tends to become numerically unstable at very high Reynolds numbers and rapidly loses accuracy at low Reynolds numbers, ultimately leading to a blow-up.

In order to address the limitations of the standard lattice BGK model and enhance its robustness and effectiveness, several alternative collision models have been proposed. The multiple-relaxation-time (MRT) model [32–34] is performed in the moment space, enabling the independent control of relaxation rates for different moments, resulting in enhanced accuracy and stability. In the cascaded model [35–38], the relaxation is formulated in the central moment space, with these moments defined in the reference frame that moves with the local fluid motion. Numerical results [35, 36] demonstrate that the cascaded model exhibits superior stability compared to the MRT model. It is noteworthy that Geier et al. [39, 40] further refined their proposed cascaded model and introduced the so-called cumulant model. Both of these models can be viewed as the MRT-type model [41], implying that they share some common drawbacks while improving model stability. The relaxation matrices in these models are controlled not only by the shear viscosity but also by the bulk viscosity and several free parameters, and a definitive approach to determining the optimal relaxation matrices is currently lacking. Moreover, these models incur substantial computational overhead. It is worth noting that the entropy lattice Boltzmann model proposed by Karlin and his coworkers [42, 43] demon-