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# Shock Wave Structure of One-Dimensional Hydro-Elastoplastic Riemann Problem with Elastic-Plastic-Fluid Phase Transition

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**Abstract.** In this work, the types of shock wave structure for hydro-elastoplastic model under compression are researched. The emphasis focuses on the theory of shock transition in the presence of elastic-plastic-fluid phase transition. As a result, in addition to the classical three-wave structure, two new shock wave patterns are found with the increase of loading strength. Several numerical tests are presented to verify the existence of the three types of wave structure.

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**Key words**: Shock wave structure, Hydro-elastoplastic model, shock transition.

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## 1 Introduction

Under some extreme conditions, solid materials can undergo elastic-plastic-fluid phase transition. For example, when meteorites splash down on rock medium, very high temperature and pressure will arise and cause surrounding rocks to change into liquid or even gaseous state. In this situation, the material strength may be ignored and the rock can be regarded as fluid. As the shock wave propagates outward, the material gradually restores the elastic-plastic characteristics of solid with temperature decreasing. In order to describe the transition process of fluid and solid naturally, researchers proposed the hydro-elastoplastic model [4,8,12–14]. This model has been successfully applied to many dynamics problems, such as explosions and high-speed impacts.

This short note researches the shock wave structure for hydro-elastoplastic model with general Grüneisen-type equation of state (EOS). This work itself is of great significance at physical level. The research result can provide guide or reference for impact tests [6]. However, our motivation is not only for its physical applications, but also for the design of numerical algorithms. Riemann problem is the essential element of computational fluid dynamics (CFD) [2], and the construction of Riemann solver is the core of designing numerical methods for fluid dynamics. Our study can provide theoretical basis for proposing exact Riemann solver or constructing approximate Riemann solvers. For example, one can develop a new Riemann solver, which takes into account the merging waves discussing in this paper. This new solver may improve the resolution of stress waves in some extreme applications, such as strong compression problem. One of the difficulties of designing such solver is how to come up with the reasonable wave structure criterion and simplification. This is our future work.

Compared with pure fluid and solid, the wave structure in the hydro-elastoplastic model is more complex due to various phase transitions. Among them, the greatest difficulty faced by us is the elastic-plastic-fluid phase transition under compression. Most of the previous works [4, 8, 13] deal with this situation with a basic assumption of three-wave structure: the leading elastic shock wave, the middle plastic shock wave and the trailing fluid shock wave. However, some researchers [1, 9, 10] pointed out that the phenomenon of merging wave may occur in the presence of strong compression for elastic-plastic model. In [6], we found and proved that when the loading intensity exceeds a certain critical value, the plastic wave can exceed the elastic wave, and these two waves merge into a more strong shock wave.

In this work, we investigate possible shock wave patterns of one-dimensional hydro-elastoplastic Riemann problem with elastic-plastic-fluid phase transition. The characters of Hugoniot curves and some basic properties of shock transition are studied. Based on these analyses, we find that there exist two critical values of shock strength, beyond which new merging waves will occur. The consequent is that there are two new types of shock structures besides the classical three-wave structure, namely a two-wave pattern and a one-wave pattern.