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Deep Neural Network for Oscillation Dynamics of Capillary Rise

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Abstract. The phenomenon of capillary rise in a tube is mainly affected by the inertia, capillary force, gravitational force, and viscous force of fluids. It has been observed that certain fluid properties and tube geometries can cause the liquid column to oscillate. In this paper, we first present a general second-order ordinary differential equation that can be used to describe the dynamic process of oscillation in terms of the tube length, tube radius, and contact angle. However, it is difficult to obtain its finding an analytical solution owing to the existence of nonlinear terms. Thus, we adopt a deep neural network (DNN) method to solve this equation for its distinct feature in function-fitting ability. Simulations are conducted to test the performance of the DNN method, and the results are found to be in good agreement with data reported in the literature. Based on a suitably designed and trained DNN, we further perform a comprehensive analysis of the rising height and moving velocity of the fluid during the oscillatory process for different combinations of tube length, tube radius, and contact angle. The results show that the DNN method provides an effective tool for studying the oscillation dynamics of capillary rise.

AMS subject classifications: 65L05, 65M99, 65Z05

Key words: Capillary rise, oscillation, deep neural network, mathematical model.

1 Introduction

Capillary rise is a classical physical problem that can be observed both in nature and our daily lives [1], such as the subsurface water flow in soil, the absorption or transport of

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water by plants, and ink spreading into cloth. From a mathematical point of view, the phenomenon of capillary rise can be depicted by some simplified or approximate models. Lucas [2] and Washburn [3] first conducted pioneering work on capillary rise, and by neglecting the inertial and gravity effects, they obtained the famous Lucas–Washburn equation in the viscous regime (or Lucas–Washburn regime), which states that the meniscus height h is proportional to the square root of time t. However, Quéré [4] pointed out that, when the tube first contacts the liquid, inertial effects must be considered. In this inertial regime, the meniscus height increases linearly over time, and oscillations can be observed when the meniscus reaches the equilibrium height (referred to as the Jurin height). Over time, more liquid will enter the tube, and so inertial and viscous effects should be considered. Bosanquet [5] conducted a theoretical study on the capillary rise in this regime, and also gave an analytical solution for the meniscus height. As the liquid column rises gradually, more liquid advances into the tube, at which point gravity cannot be neglected. Fries and Dreyer [6] derived an explicit solution for the meniscus height, and also carried out a special investigation of the transition from the inertial regime to the viscous and gravity regime. Chen et al. [7] presented a mathematical model in which the dynamic contact angle, pressure loss, and curved liquid surface in the reservoir can be analyzed during the capillary rise process. However, the effect of displaced fluid has not been considered in previous studies.

To analyze the effect of the displaced fluid on the rising process of a wetting liquid, Hultmark et al. [8] investigated the influence of the gas (displaced fluid) on the liquid imbibition process experimentally and theoretically, and derived an approximate solution for the meniscus height under certain assumptions. Walls et al. [9] performed a more comprehensive study on the effect of the displaced fluid, and obtained some theoretical solutions under different limiting cases. Note that these works only considered Newtonian fluids. Recently, Shan et al. [10] conducted some theoretical work on the capillary rise of non-Newtonian power-law fluids under the influence of the displaced fluid and the power-law index, and presented some exact solutions for different regimes and different limiting cases. They also found that, during the capillary rise of a wetting liquid, the dynamic behavior is mainly influenced by the inertial, gravity, and viscous forces, as well as the capillary effects [10].

As discussed above, in addition to the normal rising process in the Lucas–Washburn regime, there exists a stage in which the meniscus height does not rise monotonously in time, but oscillates near the Jurin height [11–14]. Additionally, the oscillation behavior depends on a critical condition relating to the inertial, gravitational, and viscous forces, beyond which the liquid column will oscillate [15, 16]. We note that different dynamic regimes of capillary rise have also been analyzed by Das and Mitra [17], who found that the Lucas–Washburn regime, inertial regime, and oscillation regime, and the transitions among them, are mainly determined by the ratio of the Ohnesorge number to the Bond number. However, in the abovementioned works, only approximate solutions were obtained under certain assumptions [10]. In general, it is difficult to derive analytical solutions to the governing equation for capillary rise due to the existence of nonlinear terms.