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Postbuckling Analysis of Shear Deformable FGM Cylindrical Shells with Porosities Subjected to Combined Mechanical Loads in Thermal Environments

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Abstract. Motivated by evident lack of studies on the nonlinear stability of porous functionally graded cylindrical shells under combined mechanical loads, this paper presents an effective analytical approach to investigate the postbuckling behavior of moderately thick functionally graded circular cylindrical shells with porosities subjected to combined action of axial compression and external pressure in thermal environments. Porosities are evenly or unevenly distributed within the functionally graded material (FGM). Due to presence of porosities, effective properties of the FGM are determined using a modified version of linear rule of mixture. Formulations are based on first order shear deformation theory taking into account von Kármán-Donnell nonlinearity. Analytical solutions of deflection and stress function are assumed to satisfy simply supported boundary conditions and Galerkin method is adopted to derive closed-form results of buckling loads and nonlinear load-deflection relations. Parametric studies are carried out to evaluate the effects of material and geometry properties, preexisting mechanical loads and temperature, distribution type and volume fraction of porosity on the postbuckling load carrying capacity of porous FGM cylindrical shells.

AMS subject classifications: 65N30, 74E30, 74G60, 74K25

Key words: Porosity, mechanical postbuckling, cylindrical shell, two-term deflection, combined loads.

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1 Introduction

Circular cylindrical shell (CCS) is a typical type of closed shell widely used in many structural applications, especially in aerospace engineering and pressure vessels. Since CCSs are frequently exposed to severe loading conditions, their stability is a problem of great importance. Functionally graded material (FGM) is an advanced composite the superior characteristics of which result from outstanding properties of constituent materials, namely metal and ceramic. Specifically, FGM possesses high stiffness and strength, excellent temperature withstanding capacity along with structural integrity. Stimulated by these features of FGM, numerous studies on static and dynamic responses of FGM structures have been carried out. Typical studies relating to the subject of the present work are reviewed in the following. Linear buckling behavior of FGM cylindrical shells under thermal and mechanical loads is the subject of some studies. Using classical shell theory (CST) and analytical solutions, linear buckling problem of thin FGM CCSs under thermal loads has been treated by Shahsiah and Eslami [1] and Wu et al. [2] adopting coupled and uncoupled forms of linear stability equations, respectively. Khazaeinejad et al. [3] used first order shear deformation theory (FSDT) and analytical solutions to study the linear buckling of FGM CCS under combined action of axial compression and lateral pressure. Based on a higher order shear deformation theory (HSDT) and an analytical approach, Bagherizadeh et al. [4] computed critical loads of FGM CCSs surrounded by an elastic medium and subjected to mechanical loads. In the above studies [1–4], linear stability equations of the CCS in the coupled form [1,3,4] or uncoupled form [2] of displacements are established using adjacent equilibrium criterion [5] and then solved employing one-term analytical solutions for simply supported boundary conditions. A buckling analysis of thin FGM CCSs under combined mechanical loads has been carried out by Huang et al. [6] using Ritz energy method and finite element method. Employing adjacent equilibrium criterion and analytical solutions, the buckling analyses of thin and shear deformable FGM CCSs with initial imperfection under thermal and compressive loads have been performed by Sun et al. [7,8]. The CST and shooting method were used in work of Wan and Li [9] to explore thermal buckling behavior of thin FGM CCSs with simply supported and clamped edges. Closed-form results of linear buckling problems for FGM conical shells and CCSs under external pressures have been presented in works of Sofiyev and coworker [10,11] adopting the FSDT and Galerkin method. Based on different versions of FSDT and finite element method, thermal buckling behaviors of FGM plates and cylindrical shells have been analyzed by Kandasamy et al. [12] and Trabelsi et al. [13].

Beside linear buckling analyses, studies of nonlinear stability of FGM CCSs under mechanical and thermal loads have been addressed. Shen [14, 15] used the CST and asymptotic solutions to analyze the postbuckling of thin FGM CCSs under axial compression and lateral pressure in thermal environments. A postbuckling analysis of shear deformable FGM CCS under combined axial and radial mechanical loads in thermal environment was presented in work of Shen and Noda [16] adopting the HSDT. Huang and