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High-Resolution Upwind Strategy for the MLS Method with Applications to Convection Dominated Diffusion Problems

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Abstract. In this paper, a new improved upwind technique for the moving least squares (MLS) method is proposed for solving convection dominated diffusion problems. The MLS approximation reconstructs a given function from randomly distributed data, which has been successfully applied to many meshless methods. To reduce spurious oscillations and improve accuracy, an upwind strategy with a limiter is designed for the MLS method. The method allows the reconstruction process to switch between different neighborhoods, several numerical examples are tested to confirm the feasibility of the proposed m ethod. For the method, the convection term is approximated on the upwind recomstruction template and a limiter is used while the diffusion term is still approximated by a central reconstruction template. Numerical experiments are conducted to show the superiority and effectiveness of the high-resolution upwind MLS with limiter under a variety of convection dominated diffusion problems, and comparisons are made to see the advantages over some other methods.

AMS subject classifications: 65M10, 65M70, 35L03

Key words: Convection dominated problems, the MLS method, upwind strategy, limiter.

1 Introduction

In recent years, a variety of numerical methods for solving partial differential equations have emerged and been widely used in many research fields [1–6, 16–29]. Among them, the MLS method [4–6] has attracted the attention of researchers with its ability to handle scattered data without using any grid. Subsequently, various improved MLS methods have been widely developed and applied [7–16]. The generalized MLS (GMLS) method [7] estimates the derivative directly from the data without any detours through the derivative of numerical flux. The adaptive moving total least squares (AMTLS)

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method [8] introduces a parameter k associated with the direction of local approximation in the EIV model. The modified MLS (MMLS) method [9] uses the error function of additional terms based on polynomial basis functions to derive MLS approximations. The recursive MLS (RMLS) method [10] solves for the optimal size of the support domain by applying an arbitrary metric value to it. The shape function obtained by the improved interpolation MLS (IIMLS) method [11] has the property of a delta function, which makes it easier for meshless methods to implement boundary conditions. The stabilized adaptive orthogonal IIMLS (SAO-IIMLS) method [12] applies weighted orthogonal basis functions to the IIMLS method to improve the quality of the obtained diagonal matrix. The augmented MLS (AMLS) [13] method uses fundamental solutions as basis functions to improve accuracy. A piece-wise moving least squares (PMLS) method [14] is proposed to reduce the computational cost, which is an optimal design with certain localized information. At present, the research of MLS-derived methods is very popular, and it is still a reliable choice for solving partial differential equations, including convection-diffusion equations and Burgers' equations, etc.

Convection-diffusion equation is a partial differential equation describing the process of material transport, which has a wide range of applications in many fields. The convection effect is only affected by the information of upstream points, and the diffusion effect is integrated with the information of surrounding points. In the numerical calculation, the diffusion term in the equation is generally calculated by the central discrete scheme with high accuracy. However, if the convection term is discretized by the central numerical scheme, its physical nature cannot be reflected, which often causes the oscillation of the numerical solution and brings difficulties to the numerical calculation within a finite time step [15–17]. For the finite difference method and other grid-based methods, an upwind type reconstruction strategy can be implemented to overcome numerical spurious oscillations, but it is more difficult for the meshless methods [18,19]. The shifting distance and the radius of the neighborhood in the upwind direction are related to the number and distribution of nodes in the support domain, the scale of the support domain and other factors [20–24]. In order to overcome such shortcomings, H. Lin et al. [22,23] proposed some upwind schemes to stabilize the convection term in the streamline direction and Zhang et al. [24] proposed the upwind strategy to approximate the convection term. However, their upwind techniques can not guarantee a highly accurate results. Recently, an effective idea was proposed by Michael et al. [25] and its effectiveness was shown by their numerical results. To approximate the solutions to the equation, they switched between two RBF interpolations of different resolutions, where the minmod limiter played a crucial role.

Motivated by the upwind techniques and the idea of the limiter, we will present a novel technique named high-resolution upwind strategy and apply it to the MLS method to solve two-dimensional time-dependent convection dominated diffusion problems at each time step. The diffusion term is still calculated by the traditional central MLS method, while the convection term is treated by selecting the upwind template to eliminate the numerical oscillation and introducing the limiter to improve the numerical ac-