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Stability of Regularized Lattice Boltzmann Model for Convection-Diffusion Equations

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Abstract. In this paper, the stability of the regularized lattice Boltzmann method (RLBM) for convection-diffusion equations is studied. First, it is shown that the evolution equation of RLBM can be transformed into two macroscopic difference schemes, one is explicit and the other is implicit. Compared with the traditional mesoscopic evolution equation, the macro evolution equation has simpler form, lower order of growth matrix, higher computational efficiency and less memory. Then, for the linear convection-diffusion equations without the source term, we use Fourier analysis to prove that the three schemes have exactly the same stability region. For the one-dimensional diffusion equation, we have proved the theoretical stability. For the D1Q3 model, D2Q5 model and D2Q9 model, we have studied the numerical stability. Our results have also been compared with related work by others, which shows they match well. Finally, we conducted some numerical tests to verify that our stability results are credible.

AMS subject classifications: 35B35, 65L08

Key words: Stability analysis, regularized lattice Boltzmann method, convection-diffusion equations.

1 Introduction

Lattice Boltzmann method (LBM) is a bottom-up mesoscopic numerical method. Compared with the traditional numerical methods, LBM is easy to deal with complex bound-

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ary, and it has the characteristics of high computational efficiency and simple programming. In the past decades, it has developed rapidly and has gained a great success in the study of complex hydrodynamic problems across a broad range of scales [1–3]. Not only that, LBM has also been widely used in solving various kinds of partial differential equations, such as Burgers equation [4, 5], Poisson equation [6] and so on.

In fact, there are many lattice Boltzmann models for the convection-diffusion equations (CDEs) [7–15], the most common are single-relaxation-time (SRT) model, two-relaxation-times (TRT) model, multiple-relaxation-times (MRT) model, regularized lattice Boltzmann method model (RLBM) [16] and block triple-relaxation-time (B-TriRT) model [17]. Among them, the MRT model includes the SRT model, and generally, the MRT model is more stable than the SRT model. The TRT model is a special case of the MRT model. In fact, the RLBM model is a TRT (MRT) model, the B-TriRT model is a MRT model.

There has been a lot of excellent work on the stability of these models. For the one-dimensional linear diffusion equation, Suga [18] derived the corresponding macroscopic difference scheme through the mesoscopic evolution equation of the SRT model, and theoretically derived the sufficient and necessary conditions for the stability of the D1Q3 model. And Suga [19] further derived the macroscopic difference scheme of the multidimensional diffusion equation SRT model, and discussed the stable region under the D2Q9, D2Q13, D3Q19 and D3Q25 models. For the isotropic and anisotropic two-dimensional convection-diffusion equations, Suga [20, 21] based on the SRT model, using three different discrete velocity models, D2Q4, D2Q5, and D2Q9, and deduced their macroscopic difference schemes and discussed their stability region numerically.I. Ginzburg et al. [10, 12, 13] studied the stability region of the TRT model, and gave the sufficient stability region of the TRT model through theoretical analysis.Wang's et al. [16] developed RLBM for the CDEs, and a large number of numerical examples are used to prove that the RLBM model is more stable and more accurate than the SRT model, and RLBM can be described by only two macroscopic quantities, but no further research has been done on the macroscopic evolution equation, and there is no relevant research on the stability of RLBM in the existing work. However, there is still a lack of systematic research on the stability region of RLBM.

In this work, we propose a new macroscopic difference scheme for the evolution equation of the RLBM based on Wang's work [16], and compared the stability of evolution equations in different schemes in Section 3. Then, we theoretically proved the stability of the one-dimensional diffusion equation in Section 4. And we research the numerical stability of the one-dimensional and two-dimensional convection-diffusion equations, and compared our results with the existing work in Section 5. Finally, we verify the effectiveness of our work through some numerical examples in Section 6.

2 RLBM for convection-diffusion equations