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Square-Mean Pseudo Almost Periodic Solutions of Infinite Class in the α-Norm under the Light of Measure Theory

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Abstract. This work concerns the existence and uniqueness of square-mean pseudo almost periodic solutions of infinite class in the α-norm. The results are obtained using analytic semigroup, fractional α-power theory and by making use of Ba-nach fixed point theory. As a result, we obtain a generalization of the work of Zab-sonre *et al.* [Partial Differential Equations and Applications: Colloquium in Honor of Hamidou Touré, Springer, 2023] in the deterministic case, without unbounded delay. Our results extend and complement many other important results in the lit-erature. Finally, a concrete example is given to illustrate the application of the main results.

AMS subject classifications: 34K14, 35B15, 34G10, 60J65

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1 Introduction

In this work, we study the existence and uniqueness of square-mean (μ,ν)-pseudo almost periodic solutions of infinite class in the α-norm for the following stochastic evolution differential equation:

$$dx(t) = [-Ax(t) + L(x_t) + f(t)]dt + g(t)dW(t), \quad t \in \mathbb{R}, \tag{1.1}$$

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where $-A : D(A) \subset H$ is the infinitesimal generator of compact analytic semigroup $(T(t))_{t\geq 0}$ on $L^2(P, H)$. The phase space $\mathcal{B}_\alpha$ defined by

$$\mathcal{B}_\alpha = \{\varphi \in \mathcal{B} : \ \varphi(\theta) \in D(A^\alpha) \ \text{ for } \ \theta \leq 0 \ \text{ and } \ A^\alpha \varphi \in \mathcal{B}\}, \quad \|\varphi\|_\alpha = \|A^\alpha \varphi\|,$$

is a subset of $\mathcal{B}$, where $A^\alpha \varphi$ is defined by $A^\alpha \varphi(\theta) = A^\alpha(\varphi(\theta))$ for $\theta \in]-\infty, 0]$ and $\mathcal{B}$ is a Banach space of functions mapping $]-\infty, 0]$ into $L^2(P, H)$ and satisfying some axioms that will be presented later. A^α is the fractional α-power of A that will be described later. For every $t \geq 0$, the history function $u_t \in \mathcal{B}_\alpha$ is defined by

$$u_t(\theta) = u(t + \theta), \quad \theta \in]-\infty, 0].$$

L is a bounded linear operator from $\mathcal{B}_\alpha$ into $L^2(P, H)$.

Here $f : \mathbb{R} \to L^2(P, H)$ and $g : \mathbb{R} \to L^2(P, H)$ are two stochastic processes and $W(t)$ is a two-sided standard Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, P, \mathcal{F}_t)$ with

$$\mathcal{F}_t = \sigma\{W(u) - W(v) | u, v \leq t\}.$$

We assume $(H, \|, \|)$ is a real separable Hilbert space and $L^2(P, H)$ is the space of all H-valued random variables x such that

$$\mathbb{E}\|x\|^2 = \int_\Omega \|x\|^2 dP < \infty.$$

Recall that stochastic modeling is crucial to many fields such as physics, engineering, economics, and social sciences. To this end, stochastic differential systems have been the subject of much research in recent years. Researchers are increasingly interested in the above mentioned quantitative and qualitative aspects of stochastic differential systems, such as existence, uniqueness, and stability. To this end, some recent contributions have been made concerning square-mean pseudo almost periodic for abstract differential equations similar to Eq. (1.1), see, for example, [4, 5, 9] and the references therein.

The aim of this work is to extend the results obtained by Zabsonre *et al.* [18], whose authors studied the Eq. (1.1) in the deterministic case. Note that some recent contributions have been made. For example, in [10], the authors studied the equation without operator L. They introduced a new concept of square-mean pseudo-almost periodic and automorphic processes using measure theory. They used the μ-ergodic process to define the spaces of μ pseudo almost periodic and automorphic processes in the square-mean sense. Moreover, they established many interesting results on these spaces, such as completeness and composition theorems. Then they studied the existence, the uniqueness and the stability of the square-mean μ-pseudo almost periodic and automorphic solutions of the stochastic evolution equation.

Recently, in [17], the authors, studied the existence and the uniqueness of the square-mean (μ, ν)-pseudo almost periodic solutions of infinite class for the stochastic evolution equation. However, to the best of the authors knowledge, the existence of