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REGULAR ARTICLE

Quantum Zeno and anti-Zeno effects in classical noise

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Abstract: Most of the existing studies are devoted to the quantum Zeno and anti-Zeno effects in open quantum systems under quantization environment; little attention has been paid to the quantum zeno dynamics behavior under classical noise. In this paper, we analyze the quantum Zeno and anti-Zeno dynamics under the random telegraph noise and the family of low-frequency noise with $1/f^{\alpha}$ spectrum. Based on qualitative analysis of effective decay rate, we find that the two kinds of classical noise under different conditions have significant influence on the Zeno and anti-Zeno dynamics. For random telegraph noise, the switching rate $\gamma > 2$ can influence the coupling strength between the system and the environment so that it can make the effective decay rate present different properties. In the case of colored noise, different coefficient and number of fluctuators N_f will make the effective decay rate change. Moreover, we also give physical explanations for these phenomena.

AMS subject classifications: 81P15, 81P20, 81S22, 81V45

Keywords: Quantum Zeno effect; Quantum anti-Zeno effect; Random telegraph noise; Colored noise.

I. INTRODUCTION

One of the appealing consequences of the quantum mechanics is that the observation unavoidably disturbs the observed system [1]. This is particularly revealed by the so-called quantum Zeno effect [2–4] which shows that rapidly repeated measurements can slow down the evolution of a quantum system. In the limiting case of continuous measurement, the evolution is expected to come to a standstill. The quantum Zeno effect is thought to be a general feature of quantum mechanics, applicable to radioactive [5] or radiative decay

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processes [6, 7]. In some cases, however, when the measurements are not frequent enough, they may actually accelerate the evolution, an effect called the anti-Zeno effect [8, 9]. It was shown in Ref. [9] that the inhibitory quantum Zeno effect may be feasible in a limited class of systems, the opposite effect–anti-Zeno effect–appears to be much more ubiquitous. The first observation of the quantum Zeno and anti-Zeno effects in an unstable system is report in Ref. [10].

The transition between quantum Zeno effect and anti-Zeno effect was studied recently. Facchi proposed that the crossover from Zeno effect to anti-Zeno effect can be specified by comparing the effective decay rate with the natural decay rate which does not involve measurement [11]. The quantum Zeno effect occurs when the effective decay rate is smaller than the natural decay rate whereas the anti-Zeno effect occurs. The transition between the quantum Zeno effect and anti-Zeno effect in a model of a damped quantum harmonic oscillator has been studied [12]. They showed that the short time behaviors of the environmentally induced decoherence plays an important role. Besides, the transition also can be observed in spin-bath models [13], and it is controlled by the system-bath dimensionless coupling strength, as well as the temperature and the energy bias between the spin states. Quantum Zeno and anti-Zeno effects on pure dephasing have been studied [14]. They showed that if the system environment coupling strength is not weak, the nontrivial evolution of the environment between measurement scan considerably alter the quantum Zeno effect and anti-Zeno effect. We note that these works focus on the Gaussian noise cases. As far as we know, little attention has been paid to the quantum Zeno and anti-Zeno effects in non-Guassian noise case.

When the physical systems are at very low temperature, experiments show that the decoherence is typically dominated by coupling with localized modes, e.g. the hopping background charges or general quantum bistable fluctuators in superconducting qubits [15–19], and nuclear spins [20]. Therefore, these localized modes could be described as finite-dimensional Hilbert spaces with finite energy cutoffs and could be mapped onto an environment of quantum system we concern. In this case different microscopic configurations of the environment leading to the same spectra may correspond to different physical phenomena. Under certain condition the knowledge of the noise spectrum is not sufficient to describe decoherence phenomena, then the noise is referred to as non-Gaussian. The role of non-Gaussian noise becomes important when the systems become smaller [21–23]. Non-Gaussian random telegraph noise commonly appears in semiconductor, metal, and superconducting devices [23]. Recently, the characteristic parameters of non-Gaussian noise are estimated by using a single quantum probe [24].The non-markovianity of random telegraph noise and decoherence induced by random telegraph noise have been investigated in Refs. [22, 23].

In this paper, we pay attention to the transition between quantum zeno effect and anti-zeno effect under two relevant classes of non-Gaussian noise: the random telegraph noise with a Lorentzian spectrum and the family of low-frequency noise with $1/f^{\alpha}$ spectrum. In the case of random telegraph noise, there is a single parameter, i.e., the switching rate γ, nevertheless colored noisewith $1/f^{\alpha}$for spectra, the tunable parameters are the exponent α and the number of fluctuators N_f Our result shows that the switching rate γ characterize markovian or non-markovian dynamics of system, as a result of it, the effective decay rate appears monotonous or oscillating, and the occurrence of transition changes depending on these parameters.

The paper is organized as follows: In Sec II, we introduce the physical model. In Sec III, we discuss the quantum Zeno and anti-Zeno effects under random telegraph noise. In Sec IV, we discuss the quantum Zeno and anti-Zeno effects under colored noise. And finally we summarize the result in Sec V.

II. PHYSICAL MODEL

We consider an initial pure state $\rho(0) = |\psi_0 >< \psi_0|$ of aquantum system evolving under the impact of classical noise. The system is described by the Hamiltonian

$$H(t) = \nu c(t)\sigma_Z. \tag{1}$$

Throughout, we work in dimensionless units and set $\hbar = 1$. In the σ_Z eigenbasis defined by $\sigma_Z|e\rangle = |e\rangle$ and $\sigma_Z|g\rangle = -|g\rangle$.Hamiltonian (1) represents a class of models of open quantum systems that describe a pure dephasing process.ν is the coupling constants between the qubit and noise respectively. $c(t)$ denotes a stochastic process of the environment noise and it may has different expressions corresponding to different kinds of noise. The density matrix for the qubit can be obtained by taking ensemble averages over the noise $c(t)$:

$$\rho(t)=\langle\rho_{st}(t)\rangle, \tag{2}$$

where $\langle...\rangle$ stands for ensemble average and the statistical density operator $\rho_{st}(t)$ is given by

$$\rho_{st}(t) = U(t)\rho(0)U^{+}(t), \tag{3}$$

where $\rho(0)$ is the initial state of probe system. The unitary operator $U(t)$ can be written as

$$U(t) = \exp[-i\int dt' H(t')]. \tag{4}$$

Then the reduced density matrix of the qubit dynamics with the time is obtained by

$$\rho(t)=\begin{pmatrix} \rho_{00}(0) & \rho_{01}(0)q(t) \\ \rho_{10}(0)q(t) & \rho_{11}(0) \end{pmatrix} \tag{5}$$

where q(t) is the noise term obeying the equation

$$q(t) = \langle e^{i\varphi(t)} \rangle, \tag{6}$$

and satisfies

$$\varphi(t) = \int_0^t c(t')dt'. \tag{7}$$

A quantum Zeno effect typically arises if one performs a series of measurements at time intervals t. We define the projective measurement applied in the quantum Zeno dynamics as $P[\circ] = M \circ M$ where with $M = |\psi_0 >< \psi_0|$. Here we take the initial state as $|\psi_0\rangle = \cos\theta|g\rangle + \sin\theta|e\rangle$. After one measurement at time τ, the survival probability of the initial state $P^{(1)}(\tau)$ can be expressed as

$$P^{(1)}(\tau) = 1 - \frac{1}{2}\sin^2(2\theta)\left[1 - P^{(1)}(\tau)\right]. \tag{8}$$

Because the survival probability $P^{(n)}(\tau)$ would be just the nth power of the *n*th power of the survival probability associated with one measurement, it is convenient to $P^{(n)}(\tau) \equiv e^{-\Gamma(\tau)t}$, with $t = n\tau$ and $1/\Gamma(\tau)$ being an effective lifetime of the initial superposition state that depends on the measurement interval τ. One then obtains

$$P^{(n)}(\tau) = \left[P^{(1)}(\tau)\right]^n = e^{-\Gamma(\tau)t}, \tag{9}$$

where $\Gamma(\tau)$ is defined as the effective decay rate. At short times an effective decay rate is identified as:

$$\Gamma(\tau) = \frac{1}{\tau}\left[1 - P^{(1)}(\tau)\right]. \tag{10}$$

In what follows we use the following qualitative definition: The quantum Zeno effect takes place when the effective decay rate decreases when τ becomes smaller, while a system for which the effective decay rate increases for smaller τ, i.e., measurements enhance the decay, shows the anti-Zeno effect. The occurrence of maxima in $\Gamma(\tau)$ is an indication for Zeno effect to anti-Zeno effect transition. This definition for the Zeno and anti-Zeno effects is therefore based on the local properties of the decay rate. While we note that this is different from the standard classification based on the fixed, natural decay.

III. RANDOM TELEGRAPH NOISE CASE

In this section, we will discuss the quantum Zeno and anti-Zeno effects under Random telegraph noise. Random telegraph noise is very common in nature, generated from a bistable fluctuator flipping between two values with a switching rate ξ. It appears in many semiconductor, metal, and superconducting devices. In order to describe a random telegraph noise, the quantity $c(t)$ in Hamiltonian flips randomly between values ± 1 with ξ.The correlation function for the process is

$$c(t, t_0) = \langle c(t) c(t_0) \rangle = e^{-2\xi|t-t_0|}. \tag{11}$$

Using Fourier transformation we can obtain the power density spectrum$S(\omega)$

$$S(\omega) = \int_{-\infty}^{+\infty} c(t, t_0)\, e^{-\omega i(t-t_0)} d(t-t_0), \tag{12}$$

where ω denotes the transition frequency of the two-level system. Simplify the equation above, we get the formula of $S(\omega)$

$$S(\omega) \propto \frac{\xi}{4\xi^2+\omega^2}. \tag{13}$$

Here we use dimensionless quantities. In particular we define the dimensionless time $\tau \equiv \nu t$ and switching rate $\gamma \equiv \xi/\nu$. Then the qubit density matrix under the random telegraph noise can expressed as

$$\rho(\tau) = \begin{pmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta\, G(\tau,\gamma) \\ \frac{1}{2}\sin 2\theta\, G(\tau,\gamma) & \sin^2\theta \end{pmatrix}, \tag{14}$$

where

$$G(\tau,\gamma) = e^{-\gamma\tau}\left(\cosh\delta\tau + \frac{\gamma}{\delta}\sinh\delta\tau\right), \tag{15}$$

with $\delta = \sqrt{\gamma^2 - 4}$. We note here that to obtain Eq. (15), we should consider the higher-order correlation functions and spectral densities. And this will bring in more new features than Gaussian noise case. Without loss of generality, we let $\theta = \pi/4$. After some calculation we can get the effective decay rate:

$$\Gamma_{RTN}(\tau) = \frac{1}{2\tau}[1 - G(\tau,\gamma)]. \tag{16}$$

According to the standard definition of the quantum Zeno effect, the behavior of the effective decay identifies the occurrence of the Zeno or anti-Zeno effects. On account of the switching rate γ characterize markovian or non-markovian dynamics of system, in other words, the threshold between the markovian and non-markovian regime corresponds to $\gamma = 2$, we discuss the effective decayrate respectively with the $\gamma > 2$and $\gamma < 2$.

We plot the effective decay rate as a function of the scaled time τ for different values of switching rate γ in**Fig.1**. From **Fig.1** we can find that the effective decay rate $\Gamma_{RTN}(\tau)$shows different properties for different values of γ. When $\gamma > 2$, according to **Fig.1(a)**, we find that the effective decay rate $\Gamma_{RTN}(\tau)$increases with increasing τ in the short time limit, after reaching a maximum, the effective decay rate reverses its qualitative dependence on τ, it begins to decreasesas τ increases.The presence of this maximum value clearly indicates the transition between quantum Zeno and anti-Zeno effects. We denote Γ_{max} as the maximum value of $\Gamma(\tau)$and τ_c as the corresponding critical point of the time interval τ. We can know that quantum Zeno effect occurs in case of $\tau < \tau_c$, while quantum anti-Zeno effect occurs in case of $\tau > \tau_c$.

Besides, from **Fig.1(a)** we can also find that when $\gamma > 2$, the value of Γ_{max} and τ_c becomes smaller with γ increase. That is, the transition between the quantum Zeno and anti-Zeno effects moves left if γ become larger,the phenomenon can be significant for experiment. And when γ is large enough, $\Gamma(\tau)$ trend to horizontal, itmeans that the quantum anti-Zeno effect is not obvious any more.

From **Fig.1(b)** we can find that when $\gamma < 2$, $\Gamma(\tau)$ ingeneral has more than one maximum. This means that the quantum Zeno effect regimes and the quantum anti-Zeno effect regimes appear alternately. This phenomenon reflects the fact that small values of γ correspond to non-Gaussian noise case. In this case high-order correlation functions play an important role in solving the model. And it means that the quantum system and the environment have a long-standing relationship, the coupling strength between the system and environment is strong. While in the case $\gamma > 2$, the effect of the high-order statistics is so small that can be disregarded, and then the characteristics of the non-Gaussianity become less pronounced, the association of system and environment is not very close, so in this case $\Gamma(\tau)$ shows just one maximum.

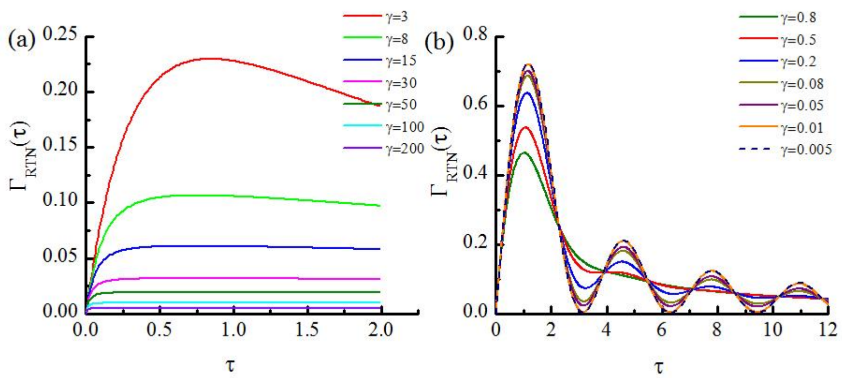

FIG. 1: The effective decay rate $\Gamma_{RTN}(\tau)$ as a function of the measurement interval τ with different γ (Color online). Fig.1(a) shows the effective decay rate $\Gamma_{RTN}(\tau)$with $\gamma > 2$, Fig.1(b) shows the case with $\gamma < 2$.

IV. COLORED NOISE CASE

Then we study the quantum Zeno and anti-Zeno effects under colored noise. Noise characterized by $1/f^{\alpha}$ spectrum is a universal noise in nature. They can be found when the environment can be expressed as a collection of N_f random bistable fluctuators, with $N_f \geq 1$, and their typical values of the coefficient α range between 0.5 and 2. Every fluctuator has an unknown switching rate, taken from the ensemble $\{\gamma_i, p_{\alpha}(\gamma_i)\}$, where the probability distribution is:

$$p_{\alpha}(\gamma) = \begin{cases} \frac{1}{\gamma \ln(\gamma_2/\gamma_1)} & (\alpha = 1) \\ \frac{\alpha-1}{\gamma^{\alpha}}\left[\frac{(\gamma_1\gamma_2)^{\alpha-1}}{\gamma_2{}^{\alpha-1}-\gamma_1{}^{\alpha-1}}\right] & (\alpha \neq 1) \end{cases}, \tag{17}$$

where γ_1 and γ_2 are the smallest and the biggest switching rate considered. The coefficient $c(t)$ is a linear superposition of N_f random bistable fluctuators,

$$c(t) = \sum c_i(t). \tag{18}$$

where $c_i(t)$ describes a stochastic telegraphic process with a Lorentzian spectrum. The qubit density matrix can thus be written as:

$$\rho(\tau) = \begin{pmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta\, \Lambda(\tau, \alpha, N_f) \\ \frac{1}{2}\sin 2\theta\, \Lambda(\tau, \alpha, N_f) & \sin^2\theta \end{pmatrix}. \tag{19}$$

where

$$\Lambda(\tau, \alpha, N_f) = [\int_{\gamma_1}^{\gamma_2} G(\tau, \gamma) p_{\alpha}(\gamma)]^{N_f}. \tag{20}$$

As above we let $\theta = \pi/4$. After some calculation we can get the effective decay rate:

$$\Gamma_{CN}(\tau) = \frac{1}{2\tau}[1 - \Lambda(\tau, \alpha, N_f)]. \tag{21}$$

Giving Eq. (21) we can study the influence of colored noise on the quantum Zeno dynamics and the quantum anti-Zeno dynamics. In **Fig. 2**, we give the effective decay rate $\Gamma_{CN}(\tau)$ as a function of the scaled time τ for different values of α. Here we take the number of fluctuators $N_f = 10$ for Fig. 2 (a) and $N_f = 40$ for **Fig. 2 (b)** and $\gamma \in (10^{-4}, 10^{4})$.

From the **Fig. 2**, we can find that $\Gamma_{CN}(\tau)$ is dramatically influenced by α, for the bigger α, it is obvious that the effective decay rate is characterized by pronounced oscillations. And the oscillations are less noticeable even fade away with α decreases.

This can be explained as follow: From Eq. (17) we can find that for the smaller values of

α fluctuators with larger switching rate occupy the dominant probability. As with the random telegraph noise in Fig. 1, the dynamics of the system is generally Gaussian with less noticeable oscillations. The larger values of α, corresponding to the lower frequency of noise spectrum, the fluctuators with smaller switching rate take the larger probability so the dynamics is non-Gaussian. Meanwhile, the lower the frequency, the larger the enhancement of the oscillations.

In **Fig. 3** we give the effective decay rate as a function of the scaled time τ for different values of N_f. Here we choose $\alpha = 1.2$. From **Fig. 3**, we can find that $\Gamma_{CN}(\tau)$ is dramatically influenced by the number of fluctuators N_f. For small N_f, the coupling strength between the system and the environment is enhanced, and the memory of the environment is obvious, $\Gamma_{CN}(\tau)$ shows oscillating behavior which indicates the quantum Zeno effect regimes and the quantum anti-Zeno effect regimes appears alternately. When the value of N_f increases, the coupling strength between the quantum system and the environment decreases, the correlation decreases, and the dynamic evolution of the system gradually exhibits markovian properties, so the oscillations are less and less noticeable. When the value of N_f is sufficiently large,the effective decay rate has no oscillation anymore, as the case of $N_f = 50$, which means if N_f is large enough, that is the number of fluctuators is enough, the coupling strength between the system and the environment is weak, the effective decay rate $\Gamma_{CN}(\tau)$ presents Gaussian characteristics with the measurement interval τ.

This can be explained by the central limit theorem. In probability theory, the central limit theorem establishes that, when independent random variables are added, their sum tends toward a Gaussian distribution even if the original variables themselves are not normally distributed. For a small number of fluctuators and a noise spectrum dominated by low frequencies indicates Non-Gaussian. This is why the oscillations of $\Gamma_{CN}(\tau)$ are noticeable for small N_f When N_f becomes large enough, the noise become Gaussian one. Then the oscillations of $\Gamma_{CN}(\tau)$ become less noticeable.

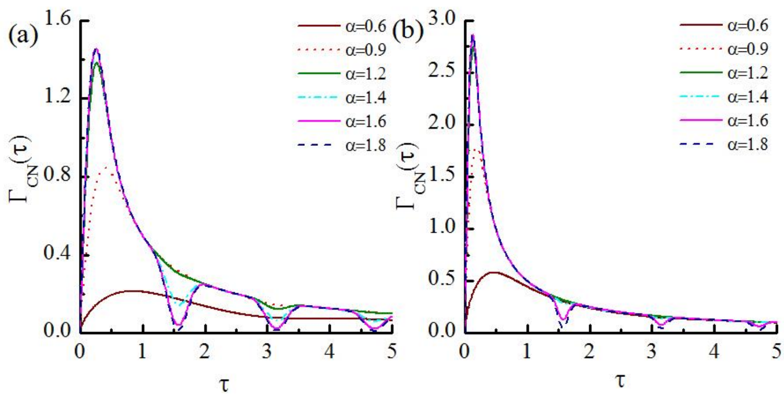

FIG. 2: The effective decay rate $\Gamma_{CN}(\tau)$ for a qubit subject to $1/f^{\alpha}$ noise generated by fixed random fluctuators for different values of α (Color online). Fig. 2 (a) shows $N_f = 10$, Fig. 2 (b) shows $N_f = 40$.

V. CONCLUSION

In summary, we have investigated the quantum Zeno and anti-Zeno dynamics under the non-Gaussian noise, which are characterized by the random telegraph noise and the family of low-frequency noise with $1/f^{\alpha}$ spectrum. First we get the explicit formula of the effective decay rate under the two typical non-Gaussian noise. Then we show that quantum Zeno and anti-Zeno dynamics is greatly influenced by the classical noise. Specifically, for random telegraph noise, when the switching rate $\gamma > 2$,the coupling strength between the system and the environment is small, the dynamic behavior of the system presents the markovian characteristics, the transition between Zeno effect and anti-Zeno effect occurs when the measurement time interval increases. On the contrary, when $\gamma < 2$, the coupling strength between the system and the environment is large, the dynamic behavior of the system presents the non-markovian characteristics, the effective decay rate shows damped oscillation as the measurement time interval increases, which indicates that the quantum Zeno effect and anti-Zeno effect appear alternately. In the case of colored noise, different coefficient α and number of fluctuators N_f will make the effective decay rate change. We find that for fixed N_f, when α is small, the oscillation of effective decay rate is not obvious, which indicates that the quantum anti-Zeno effect is not obvious, when α increases, the oscillation of effective decay rate begin to appear and gradually obvious, which indicates that quantum anti-Zeno effect appear significantly and quantum Zeno effect and anti-Zeno effect appear alternately with the measurement time interval increases. For fixed α, the coupling strength between the system and the environment weakens with the increase of N_f, the oscillation of effective decay rate disappear gradually. This is because when the N_f increases, a number of independent noise and collective behavior will show Gaussian features according to the

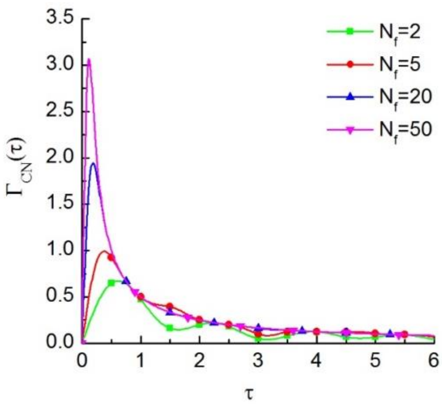

FIG. 3: (Color online) For the fixed value of $\alpha = 1.2$, theeffective decay rate $\Gamma_{CN}(\tau)$ subject to $1/f^{\alpha}$ noise as a function ofdifferent number of fluctuators.

central limit theorem of probability theory. Therefore, with the increase of N_f, the evolution of the system gradually changed from non-Gaussian to Gaussian. All these phenomena help us to better understand the behavior of quantum Zeno and quantum anti-Zeno dynamics, so as to provide a good foundation for understanding the evolution of quantum system under the classical noise.

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