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Lattice Boltzmann Modeling of Non-Newtonian Fluid Flows Under Non-Linear Slip Velocity Boundary Conditions in Microchannels

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Abstract. Fluid flows in microfluidic devices are often characterized by non-Newtonian rheology with non-linear wall slip behavior also observed. This work solves this problem class with the lattice Boltzmann method (LBM), proposing new advanced boundary scheme formulations to model the joint contribution of non-linear rheology and non-linear wall slip laws in application to microchannels of planar and circular cross-section. The non-linear stress-strain-rate relationship of the microflow is described by a generalized Newtonian model where the viscosity function follows the Sisko model. To guarantee that LBM steady-state solutions are not contaminated by numerical errors that depend on the viscosity local value, the two-relaxation-time (TRT) collision is adopted. The fluid-wall accommodation model considers different slip laws, such as the Navier linear, Navier non-linear, empirical asymptotic and Hatzikiriakos slip laws. They are transcribed into the LBM framework by adapting the local second-order boundary (LSOB) scheme strategy to this problem class. Theoretical and numerical analyses developed for a steady and slow viscous fluid within 2D slit and 3D circular pipe channels demonstrate the parabolic level of accuracy of the developed LSOB scheme throughout the considered non-linear slip and non-Newtonian models.

AMS subject classifications: 76-04, 76P05, 65Z05
Key words: Lattice Boltzmann method, two-relaxation-time scheme, slip velocity boundary conditions, non-Newtonian fluids.

1 Introduction

Boundary slip phenomenon has received growing attention in both gas [1–3, 7] and liquid [4–7] flows in most part motivated by the development of micro- and nano-scale

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technologies [8]. With the scale reduction, the interaction between the fluid and the solid wall starts to exhibit physical mechanisms that deviate from the well-established no-slip velocity boundary condition [9] and that affects both gas and liquid flows.

Gas slip is typically promoted by fluid rarefaction. This property is quantified by the flow Knudsen number $\mathrm{Kn}=\lambda/L$, where λ is the gas mean-free-path (a microscopic length scale) and L is a characteristic dimension of the flow domain (a macroscopic length scale). Commonly, the slip flow regime is defined at $0.001 \leq \mathrm{Kn} \leq 0.1$, where the application of conventional hydrodynamics equations remains valid, but the no-slip velocity condition needs to be replaced by one that accounts for slip effects [10], e.g., as given by the wall slip model proposed by Maxwell [11]. When gas rarefaction is increased over $\mathrm{Kn} > 0.1$, besides the wall slip effect, new flow features need to be considered, most notably the Knudsen layers, which further modify the gas flow topology [10]. The macroscopic description of such Knudsen layers is often performed through a non-linear stress-strain-rate relationship, whose mathematical formulation is similar to that of a non-Newtonian fluid [2,3,12]. Moreover, in the presence of high shear rates, the fluid-solid interaction model of the rarefied gas also needs to take into account the non-linear nature of the underlying physics. This process is best described by non-linear slip velocity boundary conditions [7]. The inclusion of the combined effects between non-Newtonian rheology and non-linear slip laws is therefore crucial for the accurate macroscopic modelling of gaseous flows at moderate Kn regimes. These phenomena are frequently encountered in microfluidic gas flow applications, such as microfluidic gas sensors or actuators and micro-propulsion devices [13].

Liquid slip can be found in a wide variety of physical instances. Applications range from the transport of water in tight sandstones and of oil in shale matrices [14] to low drag hydrophobic surfaces [15], or industrial appliances related to polymer processes [16]. For these latter, the interplay between fluid rheology and boundary accommodation is even richer as it gives rise to different types and mechanisms of wall slip phenomena [4,16]. This work intends to advance the numerical modeling of such physical phenomena, which in the case of liquids are characterized by the dimensionless slip length $\zeta = b/L$ that quantifies the ratio between the slip length b and the flow characteristic dimension L [7]. In fact, there is large experimental evidence suggesting that in the aforementioned application fields the liquid substances flowing inside narrow pores or in tiny capillaries are more accurately treated as "dense molecular fluids" [6]. At macroscopic level, these are best described by continuum flow models that simultaneously consider non-Newtonian rheological laws with non-linear slip boundary conditions [7,16].

Unfortunately, analytical solutions to this slip flow problem class are scarce, even at the simplest flow settings [16,17]. As a result, studies in this field tend to become only accessible through numerical simulations. For this purpose, most computational efforts have been devoted to the finite element method (FEM) [18,19] or the finite volume method (FVM) [20,21] as numerical techniques. However, considering the appealing numerical characteristics of the lattice Boltzmann method (LBM) as alternative computational fluid dynamics (CFD) technique, particularly in the modeling of complex fluid