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Application of MUSIC Algorithm for Object Localization Without Diagonal Elements of Multi-Static Response Matrix

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Abstract. Generally, to apply the MUltiple SIgnal Classification (MUSIC) algorithm for the rapid imaging of small objects, complete elements of the multi-static response (MSR) matrix must be collected. However, in some real-world applications in microwave imaging, diagonal elements of the MSR matrix are unknown. Nevertheless, it is possible to obtain imaging results using a traditional approach but theoretical reason of the applicability has not been investigated yet. In this paper, we consider the application of MUSIC for a fast identification of small objects from collected MSR matrix in both transverse magnetic (TM) and transverse electric (TE) polarizations. In order to examine the applicability, fundamental limitation, and various properties of MUSIC, we establish mathematical structure of the three imaging functions and explore that the main factors of the imaging functions are Bessel function of order zero, one, and two. The established structures demonstrate why the existence and location of small objects can be retrieved via MUSIC without the diagonal elements of the MSR matrix. Results of numerical simulations with noise-corrupted synthetic data are also provided to support the identified structures.

AMS subject classifications: 78A46

Key words: MUltiple SIgnal Classification (MUSIC), multi-static response (MSR) matrix, Bessel functions, numerical simulations.

1 Introduction

Time-harmonic inverse scattering problems for the retrieval of a two-dimensional small objects in transverse magnetic (TM) polarization (or permittivity contrast case) and transverse electric (TE) polarization (or permeability contrast case) have been considered in

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various researches [3,9,12,15,19,35]. The principle of retrieving unknown targets is based on the Newton iteration method (i.e., determining the shape of the objects), which minimizes the discrepancy function between the measured far-field patterns in the presence of true and man-made targets. Various techniques for reconstructing the shape of targets have also been developed, including the Newton or Gauss-Newton methods [33, 54], level-set strategy [21,48], bifocusing method [27,51], factorization method [31,34], potential drop method [25], inverse Fourier transform [2], migration techniques [6, 42], topological derivative [4, 37], direct sampling method [26, 41], and linear sampling method [16,32].

The MUltiple SIgnal Classification (MUSIC) algorithm has been successfully used for imaging arbitrary shaped targets. For example, identification of two- and three-dimensional small targets [8,17], retrieving small targets completely embedded in a half-space [7, 24], detecting internal corrosion [10], damage diagnosis on complex aircraft structures [13], reconstruction of thin objects [38], perfectly conducting cracks [11], and extended targets [5], radar imaging [36], and biomedical imaging [50]. Throughout various researches, it has been confirmed that MUSIC is a fast, stable, and effective imaging technique. Furthermore, MUSIC can be extended in a straightforward fashion to the case of multiple non-overlapping objects. Recently, by establishing relationships with Bessel functions of integer order, various intrinsic properties of MUSIC in full- and limited-view, and limited-aperture inverse scattering problems have been revealed [1,28,30,44,46].

In several studies, the MUSIC algorithm has been applied when one can use the complete elements of a multi-static response (MSR) matrix whose elements are measured scattered field or far-field pattern. However, under certain configurations, the diagonal elements of an MSR matrix cannot be handled. For example, it is very hard to simultaneously transmit and receive the signal in microwave imaging (see [14, 43, 52] for instance) so that the assumption that all elements of the MSR matrix are available cannot be used. This is the reason of the development of bistatic imaging technique to overcome intrinsic limitation of monostatic imaging, refer to [18, 20, 29]. Fortunately, the shape of objects can still be obtained via MUSIC without diagonal elements of MSR matrix. This fact can be examined through various numerical simulation; however, the theoretical reasons for its applicability have not been investigated. This provides a stimulus for analyzing the MUSIC algorithm without the diagonal elements of an MSR matrix.

In this study, we consider the MUSIC algorithm for imaging two-dimensional small object in TM and TE polarization from MSR matrix when the diagonal elements cannot be handled. In order to show the feasibility, we carefully investigate the mathematical structure of a MUSIC-type imaging function by identifying a connection with the Bessel function of integer order of the first kind. This is based on the asymptotic expansion formula in the presence of small object in TM and TE polarizations, refer to [9]. The investigated structure explains why the location of objects can be obtained via MUSIC in both TM and TE polarizations, and it reveals the undiscovered properties of MUSIC. In order to support the theoretical results, simulation results with synthetic data polluted by random noise are exhibited.