

A Well-Balanced Lattice Boltzmann Model for Binary Fluids Based on the Incompressible Phase-Field Theory

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Abstract. Spurious velocities arising from the imperfect offset of the undesired term at the discrete level are frequently observed in numerical simulations of equilibrium multiphase flow systems using the lattice Boltzmann equation (LBE) method. To capture the physical equilibrium state of two-phase fluid systems and eliminate spurious velocities, a well-balanced LBE model based on the incompressible phase-field theory is developed. In this model, the equilibrium distribution function for the Cahn-Hilliard (CH) equation is designed by treating the convection term as a source to avoid the introduction of undesired terms, enabling achievement of possible discrete force balance. Furthermore, this approach allows for the attainment of a divergence-free velocity field, effectively mitigating the impact of artificial compression effects and enhancing numerical stability. Numerical tests, including a flat interface problem, a stationary droplet, and the coalescence of two droplets, demonstrate the well-balanced properties and improvements in the stability of the present model.

AMS subject classifications: 82B40, 76M28, 76T10

Key words: Well-balanced scheme, lattice Boltzmann method, phase-field method, Cahn-Hilliard equation.

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1 Introduction

Multiphase flows are frequently encountered in industrial operations and engineering applications. The description and prediction of the multiphase flow are difficult due to the complex interfacial behavior over a wide range of length and time scales [1, 2]. In order to comprehend them, numerous numerical simulation methods have been developed and have proven to be extremely effective [3–11]. Among them, numerical schemes [12–15] based on the kinetic theory have been emerging as an appealing methodology in recent years, since they bridge the gap between macroscopic descriptions of multiphase dynamics and microscopic intermolecular interactions. In particular, the lattice Boltzmann equation (LBE) method has received particular attention for its concise and intuitive way of representing intermolecular interactions [16–20]. Although significant advances have been achieved in the development of multiphase LBE methods, some difficulties such as spurious velocity (SV) and numerical instability for systems with high density ratio and large viscosity ratio still exist [21]. SV is a phenomenon emerging in the vicinity of the phase interface. Theoretically, when a two-phase system is in equilibrium, the chemical potential of the system should remain constant, and the velocity field should be zero. However, the existing literature has indicated that the velocity near the interface cannot be completely eliminated, which is attributed to numerical reasons and is considered non-physical [22]. It is evident that the presence of SV could produce inaccurate density properties and may lead to unphysical phenomena in some situations, causing misunderstanding of the real physics.

Over the past few decades, significant efforts have been dedicated to identifying the underlying cause of SV and mitigating its impact [23, 24]. Lee and Fisher [25] found that the spurious velocities can be effectively minimized by utilizing the potential form of surface tension alongside the isotropic finite difference scheme in the context of the free energy LBE method. Cristea and Sofonea [26] proposed that the use of the first-order upwind scheme for computing space derivatives in the evolution equation can lead to SV in the finite-difference LB equation. To address this issue, they introduced a correction force term designed to eliminate it. Subsequently, Guo et al. [27] concluded through rigorous mathematical analysis that in the free-energy based LB equation model the emergence of SV is a result of the imbalance between the surface tension force and the gradient of ideal gas pressure at a discrete level. Then Lou and Guo [28] introduced a Lax-Wendroff-type LB equation to mitigate the impact of the above imbalance, which allows control of the magnitude of SV by modifying the Courant-Friedrichs-Lewy (CFL) number. Although the above efforts are helpful for understanding the SV and reduce the magnitude, it is still difficult to completely eliminate SV. Most recently, Guo et al. [29] proposed a well-balanced (WB) LBE model, where the equilibrium distribution function is redefined such that an artificial pressure is included in its second-order moment, instead of the ideal gas pressure, and the modified force does not contain the gradient of ideal gas pressure which is present in the standard free-energy LBE model. As a result, the SV can be effectively eliminated to machine accuracy and the consistent interface profile and bulk densities