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Reconstruction of a Locally Rough Interface and the Embedded Obstacle with the Reverse Time Migration

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Abstract. Consider the inverse problem of time-harmonic acoustic scattering by an unbounded locally rough interface with bounded obstacles embedded in the lower half-space. An extended reverse time migration (RTM) is proposed to simultaneously reconstruct the locally rough interface and embedded obstacles. By constructing a modified Helmholtz-Kirchhoff identity associated with a planar interface and a mixed reciprocity relation, we propose two new imaging functionals with using both the near-field and far-field measurements. It is shown that the imaging functionals always peak on the local perturbation of the interface and the embedded obstacle. Thus, the two imaging functional can be used to reconstruct the location and shape of the rough surface and the embedded obstacle. Numerical examples are presented to demonstrate the effectiveness of the method.

AMS subject classifications: 35R30, 65N21

Key words: Inverse acoustic scattering, locally rough interfaces, embedded obstacles, reverse time migration.

1 Introduction

This paper concerns the two-dimensional inverse problem of time-harmonic acoustic scattering by a locally rough interface with an embedded obstacle in the lower half-space. Given the incident wave, the direct scattering problem is to determine the distribution of the scattered wave; while the inverse scattering problem aims to recover the locally rough

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interface as well as the embedded obstacle from the measured scattered wave in a certain domain. These problems were motivated by significant applications in diverse scientific areas such as medical imaging [1] and exploration geophysics [3].

The main difficulty of the rough surface scattering problem is the unboundedness of the rough surface, which makes the related integral operators are non-compact and so the classical Fredholm alternative is not available. Based on a generalized Fredholm theory [23, 24], the well-posedness of the direct scattering problem by rough surfaces has been established in [19–21, 43, 49] by the integral equation method. In addition, we refer to [18, 34, 44–46, 51] for the well-posedness of the direct rough surface scattering problem. Different from all previous works, if the rough surface is a local perturbation of a planar surface, by introducing a special locally rough surface, the scattering problem can be transformed into an equivalent integral equation defined in a bounded domain, for which the well-posedness follows from the classical Fredholm alternative, for details we refer to [25, 36] for the scattering by the locally rough surface and [37, 47] for the scattering by the locally rough interface with an embedded obstacle. For inverse problems, the reference [47] has established a global uniqueness which shows that the locally rough interface, the embedded obstacle and the wave number in the lower half-space can be uniquely determined by means of near-field measurements above the interface. Based on this uniqueness, a modified linear sampling method has been developed in [37] to solve the inverse problem of simultaneously reconstructing the locally rough interface and the embedded obstacle. However, it is worth pointing out that the linear sampling method in [37] is sensitive to the noise. For the inverse problem, there exists a variety of numerical algorithms. For the inverse scattering by a planar surface with buried objects, we refer to the MUSIC-type scheme [2], the asymptotic factorization method [26], the sampling method [27], and the direct imaging algorithm [32,38,40]. For the inverse scattering by rough surfaces, we refer to iterative algorithms [7–9,22], the algorithm based on the transformed field expansion [5,6], the factorization method [28], the singular source method [31], the direct imaging method [41,42], and linear sampling methods [25,36,50].

The RTM method is a popular sample-type method which has been extensively applied in seismic imaging [4] and exploration geophysics [3,4,10]. This method first back-propagates the complex conjugated data into the background medium and then computes the cross-correlation between the back-propagated field and the incident field to output the imaging indicator, which has different behaviors when the sampling point is near the scatterer and far away from the scatterer. Based on this property, it is efficient, stable, and robust to noise. The key point of the RTM method is to establish the related Helmholtz-Kirchhoff identity, which plays a crucial role in the analysis of the indicator. For the inverse obstacle scattering problem, the mathematical justification of the RTM method has been proved rigorously in [11], which is based on an usual Helmholtz-Kirchhoff identity associated with the fundamental solution of the Helmholtz operator in the free space. The results in [11] has been extended to [12–17, 29] to solve some other inverse obstacle scattering problems. However, there are few results for the RTM method to recover unbounded rough surfaces since the usual Helmholtz-Kirchhoff