

A Genuinely Two-Dimensional Approximate Riemann Solver with Stress Continuity for Hypo-Elastic Solids

Zhiqiang Zeng¹, Kui Cao¹, Chengliang Feng¹, Yibo Wang¹ and Tiegang Liu^{1,*}

¹ LMIB and School of Mathematical Sciences, Beihang University, Beijing, 100191, China.

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Abstract. The inability to maintain stress continuity across a contact discontinuity is a well-known limitation of some Godunov-type methods developed for gas when directly employed for hypo-elastic solid simulations. Interestingly, this drawback persists in multi-dimensional computations, even when a genuinely multi-dimensional approximate Riemann solver is utilized. To address this challenge, a genuinely two-dimensional Riemann solver is constructed with the enforcement of stress continuity. Subsequently, a path has been constructed by using the present one-dimensional approximate Riemann solver which ensures the stress continuity. Based upon the established path, a discretization method for stress equation is developed by utilizing the path-conservative DLM (Dal Maso, LeFloch, and Murat) approach. Numerical tests demonstrate that the proposed approximate Riemann solver effectively preserves stress continuity, thereby eliminating nonphysical numerical oscillations.

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1 Introduction

In recent decades, researchers have developed various elastic models to simulate the mechanical behaviors of elastic materials [4, 5, 13, 14, 17, 29, 39, 42]. Among those models, the

*Corresponding author. Email addresses: liutg@buaa.edu.cn (T. Liu), zhiqiangzeng2018@buaa.edu.cn (Z. Zeng), caokui@buaa.edu.cn (K. Cao), charlon.feng@buaa.edu.cn (C. Feng), yibowang@buaa.edu.cn (Y. Wang)

hypo-elastic model has exhibited promising results that closely match the mechanical behavior of metal materials in engineering computations [1,26]. For that model, a discrete scheme was constructed by Howell and Ball [15] through the separation of the conservation part of mass, moment, and energy from the non-conservative part of the stress model. Furthermore, they decomposed the stress tensor into hydrostatic pressure and deviatoric stress tensor. Consequently, the conservation system only accounted for the influence of hydrostatic pressure, resulting in the inability to maintain the stress continuity at the contact discontinuity [45]. Similar issues may also arise in the Harten-Lax-van Leer-contact (HLLC) approximation Riemann solver [10].

To tackle the above problem, researchers [6,9,11,12,19,20,24,38,45,47] have introduced various ways of considering stress continuity in the exact/approximate solver for the one-dimensional Riemann problem of solids. The above-mentioned one-dimensional works had been also extended to two-dimensional elastic solids based on the dimension-splitting method. The requirements of stress continuity were considered separately in the x - and y -directions in those studies [7,8,18,26,34,41]. Although one-dimensional Riemann solvers have been the cornerstone of constructing multi-dimensional solid solvers, some researchers have always believed that one-dimensional exact or approximate Riemann solvers might be unable to fully take into account the multi-dimensional physical characteristics in multi-dimensional problems, and genuinely multi-dimensional Riemann solvers should be employed in the presence of strong shear stress. Compared with the dimensionality splitting, the genuinely multidimensional solvers of fluids did demonstrate relevant advantages for gaseous flows [2,3,16,33]. However, similar work for elastic-plastic solids is seldom available.

Compared with compressible fluids, multi-dimensional elastic-plastic solids are governed by conservation equations (mass, momentum, and energy equations) along with non-conservative stress equations. The quest for an exact solution to the multi-dimensional Riemann problem within this framework remains an ongoing challenge. Hence, constructing approximate solvers for multi-dimensional Riemann problems in elastic-plastic solids becomes important in practical scenarios. Building upon what has been developed in fluids [2,3,16,33], one viable approach is to use conserved components to construct appropriate approximate solvers for multi-dimensional Riemann problems in solids. Related research includes genuinely two-dimensional HLL-type approximate Riemann solver [46,49]. It is observed that the approximate solver is unable to uphold stress continuity. Such limitation gives rise to unphysical numerical oscillations in the computed results when certain initial conditions are considered. Therefore, it becomes important to develop an appropriate multi-dimensional approximate solver that considers the essential physical requirements of solids, particularly stress continuity in the vicinity of contact discontinuity.

To acquire the Goudunov-type discretization method for the governing equations of solids, a suitable discretization method for the non-conservative stress equations is necessary. One existing approach is to employ Howell and Ball's method [15]; however, numerical findings indicate an erroneous estimation of stress shock wave intensity in