

Vortex Spline Method for Two-Dimensional Rayleigh-Taylor Interfaces Between Inviscid and Perfect Dielectric Fluids with External Electric Fields

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Abstract. Interfacial fluid instabilities are widespread in industrial applications. They often lead to mixing between different fluids and play an important role in industrial processes. How to control such interfacial instabilities between dielectric fluids by external electric fields have been actively explored. To understand the effects of external electric fields on the unstable interface and precisely control it, a numerical method capable of providing accurate results is indispensable and highly desired. In this paper, we present a numerical method for systems containing unstable material interfaces between incompressible, inviscid and perfect dielectric fluids in the presence of gravity and external electric fields in two dimensions. We extend the formulation of vortex sheets in the literature from hydrodynamics to electrohydrodynamics, and derive a numerical method in which the computation of both velocity and electric field is only conducted at the one-dimensional material interface. Non-uniform one-dimensional meshes are used to represent the interface shape, which captures the dominant features with fewer nodes. High-order regularization for the Birkhoff-Rott integral in the literature [1] is implemented to control the numerical instabilities. We implement a dynamic mesh adjustment algorithm to further improve the efficiency and robustness of numerical solutions. Validation studies on the convergence and accuracy are conducted.

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1 Introduction

Interfacial flows between different fluids occur in a wide variety of natural phenomena and industrial applications. Under the effects of external forces, the material interface can become unstable and exhibit complicated dynamics. One of the classical examples is the instability that occurs when a light fluid supports a heavy fluid in a gravitational or acceleration field [2,3], namely the Rayleigh-Taylor instability (RTI). Small disturbances at an unstable Rayleigh-Taylor (RT) interface grow into large structures, and the fluids penetrate into each other. Since interfacial mixing in this process plays a key role in many engineering applications, it is essential to explore the approaches to controlling such an interface. One such approach is applying an external electric field to the interface between dielectric fluids [4–8]. To precisely control the evolution of the material interface, one needs to understand the dynamics of the material interface and investigate the various effects that the external electric field exerts on the interface. For this purpose, it is important and indispensable to conduct numerical simulations for this physical system. However, the unstable nature of the physical system leads to difficulties in the stability and accuracy of numerical methods. In this paper, we present a numerical method for studying the unstable material interface between incompressible, inviscid and perfect dielectric fluids in the presence of gravity and external electric fields in two dimensions. The presented method formulates both the velocity field and the electric displacement field in terms of “vortex sheets”, which extends the vortex sheet methods from hydrodynamics in the absence of electric fields [9–15] to electrohydrodynamics which contains the interactions between gravity and electric fields.

Since the pioneering works of Rayleigh [2] and Taylor [16], the research on RTI has been active in the fields of theoretical, numerical, and experimental studies [11–13,17–38]. Following the exponential growth in the linear stage, nonlinear structures emerge at the material interface in the form of fingers. Accompanied by the nonlinear interactions and merging between adjacent fingers, the dynamics of the unstable interface eventually transitions into the turbulent stage. During this process, the evolution of the material interface, particularly the penetration between fluids that indicates the degree of interfacial mixing, is of great interest both to the researchers from the academic point of view and to the engineers aiming to control interfacial mixing in industrial applications. More details of the RTI and its applications can be found in the thorough and comprehensive reviews by Sharp [39] and Zhou [40,41].

Continuous mixing between fluids plays an important role in many industrial processes, e.g. the combustion between the fuel and the oxidizer. Therefore, precise control of unstable RT interfaces is highly desirable. For this reason, various methods for controlling the unstable material interface [42–44], including applying external electric fields to the interface between dielectric fluids, have been actively explored [3–8,45–48]. An experiment of continuum feedback control of a stable RT-type interface using electric fields was conducted by Melcher and Warren [4]. Numerical simulations on the RT-type interface in the presence of external electric fields were carried out by Cimpanu et al. [47]