

Total Variation Distance-Enhanced Selective Segmentation for Medical Images

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Abstract. In this paper, we propose an enhanced local intensity clustering energy functional designed for selective segmentation of medical images, particularly those affected by intensity inhomogeneity. The functional includes an area constraint term based on a total variation (TV) distance function derived from the single-scale Retinex output image. This TV distance function measures an unusual distance between points in the image domain and specified marker points, ensuring accurate localization of the selected objects. By combining this with local intensity clustering fitting energy and contour length regularization, the resulting minimization model achieves precisely selective segmentation and tight object wrapping. Moreover, instead of solving the Euler-Lagrange equation or using the level set method, we introduce an efficient iterative convolution-thresholding method to implement the model numerically. This method guarantees energy decay and enables faster convergence to a stable partition. Numerical experiments on some medical images demonstrate the effectiveness and efficiency of our proposed approach for selective image segmentation.

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Key words: Selective image segmentation, medical image, intensity inhomogeneity, single-scale Retinex, iterative convolution-thresholding method.

1 Introduction

Selective image segmentation is a targeted approach that extracts specific objects from an image with the help of some prescribed marker points placed near or within the regions

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of interest, which is crucial for medical imaging applications such as lesion and tumor detection. This method contrasts with global segmentation techniques, which attempt to divide the entire image into distinct regions by isolating all foreground objects that differ from the background. It also differs from interactive segmentation, where more extensive user input is required compared to the fewer inputs in selective segmentation [3, 11, 34]. By concentrating on specific areas of interest, selective segmentation achieves greater accuracy and efficiency, making it particularly valuable in medical imaging applications.

Over the past decades, variational methods have become an essential approach in image segmentation due to their ability to incorporate prior knowledge and constraints into the process [1, 27]. These methods typically fall into two main categories: edge-based and region-based models. Edge-based models rely on image gradients for their energy functionals, making them effective for images with clear boundaries. For instance, Kass et al. [16] introduced the snake model, which uses internal energy for contour smoothness and external energy to drive the contour toward boundaries, but it requires the initial contour to be near the target and struggles with handling topological changes. Caselles et al. [4] addressed this issue with the geodesic active contour model, allowing contours to naturally split and merge, enabling the detection of multiple objects. On the other hand, region-based models use statistical information from image regions and often employ the level set method to minimize energy functionals. The typical global region-based models include the Mumford-Shah model [29], the Chan-Vese model [5], and the multi-phase model [37]. While effective for binary or multi-phase images, these models can struggle with intensity inhomogeneity due to their homogeneity assumption within each region. Afterward, to overcome this issue, many enhanced region-based methods have been developed to incorporate local statistical features into the energy functional, improving segmentation accuracy in such complex images [15, 19, 20, 22, 25, 33, 39, 42–47].

With the additional help of prescribed marker points, variational selective segmentation models aim to extract specific objects by minimizing an energy functional that evaluates segmentation quality. This functional typically includes terms for data fidelity, contour length or smoothness, and other essential criteria, guiding the segmentation process to accurately capture the desired regions. Over time, variational selective segmentation models have evolved significantly. When the marker points were placed close to the boundary of the selected region, Gout et al. [9] combined the geodesic active contour [4] with a weighted Euclidean distance function using a level set approach. Badshah and Chen [2] enhanced this by incorporating intensity constraints from the Chan-Vese model [5], promoting the segmentation of homogeneous regions. Rada and Chen [31] further improved robustness with a two-level set method. However, these models often lacked size constraints for the detected objects, leading to over- or under-segmentation depending on the initialization. To address this issue, Rada and Chen [32] introduced area constraints, while Spencer and Chen [36] reformulated their model into a convex form using convex relaxation. More recently, Liu et al. [23] proposed a two-stage convex model that incorporates a distance function-dependent weight in the data-fitting term, allowing for a smooth approximation and utilizing a thresholding procedure to extract the