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A H_N^T-Based UGKS Scheme for the Three-Temperature Radiative Transfer Equations

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Abstract. The aim of this paper is to propose an asymptotic preserving numerical scheme for the three-temperature radiative transfer equations, which couple the radiative transfer equation with the electron and ion diffusive equations. The H_N^T method is used for angular discretization, and the unified gas kinetic scheme (UGKS) is used for spatial discretization to obtain the asymptotic preserving property of the proposed scheme. Since it includes the electron-ion heat conduction, radiation-electron coupling and electron-ion coupling terms, the three-temperature model exhibits strong nonlinearity and strong coupling properties. By carefully dealing with these coupling terms implicitly and using an iteration method to solve the electron-ion diffusive system, we can adapt the H_N^T-based UGKS method for the grey radiative transfer equations to deal with this complex model. According to the asymptotic preserving property of the proposed scheme, it is not necessary in optically thick regimes to restrict the spatial step size smaller than the photon mean free path in order to approximate the solution of the three-temperature, two-temperature, and single-temperature diffusion limit equations under difference circumstances. The robustness and effectiveness of the current scheme are verified by several numerical experiments.

AMS subject classifications: 65-02, 65M08, 80M35

Key words: Radiative transfer, three-temperature model, H_N^T-method, unified gas kinetic scheme, asymptotic preserving property.

1 Introduction

The objective of this paper is to construct an asymptotic preserving numerical scheme for the three-temperature radiative transfer equations, which consist of the radiative transfer

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equation, electron diffusion equation and ion diffusion equation. These coupled equations describe the interactions and energy exchange among radiation photons, electrons and ions. They are widely used in astrophysics, high energy density physics, and inertial confinement fusion [1–4].

The three-temperature equations can accurately describe physical processes, unlike the two-temperature model that only considers the coupling of one material temperature equation with radiation. In this case, the electrons and ions in the background material are not in equilibrium, resulting in different temperatures and distinct heat conduction terms. The ions can only exchange energy with the electrons, while the electrons can also interact with radiation simultaneously.

There are numerous numerical methods on the three-temperature radiation diffusion model, where the radiation is also described by a diffusion equation. Such as in [5], the three-temperature diffusion model is linearized using Pomraning form, and the resulting linearized equation is numerically solved through an integral transformation technique and the Fourier integral. In [6, 7], the three-temperature radiative diffusion problem is solved using a monotone scheme, which is tested through multiple radiation-hydrodynamic problems. In [8], a cell-centered nonlinear finite volume scheme is proposed for the non-equilibrium three-temperature diffusion equations that satisfy the maximum principle. Several nonlinear iterative methods such as those presented in [9–11] are utilized to improve the computational efficiency in solving the three-temperature conduction equations. Additionally, several parallel methods have also been investigated in [12, 13]. In [14] the model is reconstructed to establish a convex combination scheme that satisfies the maximum principle. In the article [15], a positive-preserving finite volume scheme for the radiation diffusion equation is presented. Recently, a positive-preserving conservation Lagrangian scheme for the radiation hydrodynamics with three-temperature radiation diffusion model has been studied in [16].

Due to the strong coupling, high dimensionality and nonlinearity in the model, the numerical simulation of the three-temperature radiative transfer model has encountered some challenges, and there are limited references on numerical schemes in the literature. In [17], the implicit Monte Carlo method (IMC), developed in [18] for the two-temperature model, is extended to the three-temperature radiative transfer model, and three numerical methods are proposed to handle the coupling terms among radiation photons, electrons, and ions. The method proposed in [14] for the three-temperature diffusion model is extended in [19] to a multi-group three-temperature radiation model without the electron and ion conduction, and a decoupling method is introduced to reduce computational costs.

In the recent years, the unified gas kinetic scheme (UGKS) has been introduced to solve the rarefied gas problems [20]. UGKS can easily achieve the asymptotic preserving, and then it is extended to the linear and nonlinear radiative transfer equations [21–27]. The H_N^T method, proposed in [28, 29] for the linear and gray transfer equations, combines the advantages of the discrete ordinate method and the spherical harmonic method for the angular discretization. And it is further combined with the unified gas kinetic