

Development of Cartesian Grid Method and Local Grid Refinement for Discrete Velocity Method

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Abstract. Cartesian grid method and local grid refinement for discrete velocity method (DVM) are developed in this work, with the numerical flux of DVM constructed by semi-Implicit Richtmyer method. To implement the boundary condition, the interpolation approach is applied, where the distribution function at the fluid side of the boundary point is first approximated by interpolation method given the knowledge of the boundary point. Once the distribution function is interpolated, the reflected Maxwellian distribution and numerical flux at boundary point can be evaluated. Then the distribution function at the fluid point close to the boundary can be updated by finite difference formulation. However, the interpolation of the distribution function at the boundary point is a significant challenge as the breakdown of the upwind stencil will cause instability. To preserve the upwind stencil, the most effective approach is to perform interpolation along the characteristic lines. Moreover, the local grid refinement is introduced to reduce the computational cost for industrial application. To validate the proposed Cartesian grid method, some numerical examples are simulated. The results demonstrate the accuracy and stability of the present method for straight boundary with oblique and curved boundary, subsonic and supersonic flows.

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Key words: Cartesian grid method, discrete velocity method, rarefied flows, semi-Implicit Richtmyer method.

1 Introduction

The industrial application of non-equilibrium flow problems, such as microelectromechanical system (MEMS) [1, 2], space shuttle and re-entry vehicle [3, 4] have received

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much attention in recent years. These flow problems are usually characterized by high Knudsen number ($Kn > 0.001$), which is defined as the ratio of the mean free path of gas molecules and flow characteristic length. Under such circumstances, the continuum assumption breaks down, and some non-Newtonian phenomena, such as flows without temperature gradient [5] and Knudsen paradox [6] appear. The Boltzmann equation, which is the governing equation of gas distribution function, is typically employed to solve these issues. To date, various numerical schemes have been developed for solving the Boltzmann equation.

A popular method to solve the Boltzmann equation is the discrete velocity method (DVM) or discrete ordinate method (DOM) [3, 7–11], where the Boltzmann equation is solved in both the physical space and velocity space in the framework of finite volume method (FVM). DVM was developed by Broadwell [12, 13] and further studied by Cabannes [14], Illner [15], Kawashima [16], etc. These early methods can give accurate results for high Knudsen number flows from the upper transition to the free molecule regime. However, these methods are recognized to have difficulty to simulate continuum flows. The limitation of time step, which should be less than the particle collision time, makes these methods quite expensive for continuum flow problem. To break such limitation, the implicit method for the collision part was utilized [17, 18]. The introduction of implicit method has significantly reduced the computational cost of DVM, enabling it to simulate a shuttle-like geometry in very reasonable time [19, 20]. Nevertheless, because of the intrinsic numerical dissipation proportional to time step, the calculated results for continuous flow may not be accurate enough. To resolve the bottleneck of classical DVM, some asymptotic preserving (AP) schemes were developed. The primary AP scheme was the Unified Gas Kinetic Scheme (UGKS) proposed by Xu et al. [21]. UGKS constructs the numerical flux of DVM by the integral solution of Boltzmann equation. Later, a Discrete Unified Gas Kinetic Scheme (DUGKS) was proposed by Guo et al. [22–24], and the numerical flux is constructed by the characteristic solution of Boltzmann equation. Both UGKS and DUGKS are numerical methods that combine the advection step and collision step while meeting the asymptotic preserving [25], which makes them more efficient and accurate than the classical DVM for flows in the continuum and near-continuum regime. To further simplify the formulation of UGKS and DUGKS, a semi-Implicit Richtmyer (SIR) method [26] has been developed within the framework of Richtmyer scheme, whose interfacial flux is constructed by a finite difference formulation incorporating the relaxation term. Apart from the simplification of numerical flux, several other techniques have been also developed to reduce the computational cost of DVM. For instance, the introduction of some convergence acceleration methods, such as implicit UGKS [27, 28] and general synthetic iterative scheme (GSIS) [29, 30], have increased the computational efficiency by an order of magnitude. Moreover, Yang et al. [31] combined the memory reduction technique and implicit DVM, which helps to reduce the memory required for steady-state problem to the order of magnitude as macroscopic equations. Recently, a DVM-ALE method has been developed for solving kinetic equations with moving and deforming mesh [32]. Combining parallel computing techniques, the DVM-ALE method