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New 3D Benchmark for CFD-Codes Based on Analytical Solution of Spherically-Symmetric Gas Free Expansion

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Abstract. The article presents for the first time the use of a known solution to the problem of gas expansion into a vacuum as a test for the implementation of numerical methods. An analytical solution of the problem of gas ball expanding into vacuum is written out in an explicit form. The solution is designed for benchmarking of three-dimensional gas dynamics solvers. The test makes it possible to evaluate a spherical symmetry of a numerical solution when using a Cartesian or cylindrical coordinate system. Moreover, the actual order of approximation of an implemented numerical algorithm, and the accuracy of reproducing the solution on a moving free boundary can be estimated. We present the results of numerical solution of the problem by using three-dimensional Smoothed Particle Hydrodynamics (SPH) simulations. The simulation is performed for two methods of computing density in SPH. The algorithm for computing the dynamics of particles at gas-vacuum sharp interface is described in detail.

AMS subject classifications: 35A09, 76N15, 65N22

Key words: CFD test, benchmarking problem, gas free expansion, Nemchinov-Dyson problem, CFD verification test, Smoothed Particle Hydrodynamics (SPH).

1 Introduction

Computational Fluid Dynamics (CFD) is a powerful tool to study natural phenomena and engineering devices. Reliable CFD-solvers undergoes thorough verification on prob-

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lems that have analytical or reference solution [1,2] and are aligned with physical formulation of problems that will be solved using computer simulation.

There is a classical set of model problems for a compressible inviscid gas that are used to verify numerical codes and to study the properties of new methods (for example, see [3–5]). The solutions of the Riemann problem are used as reference for one-dimensional flows. Particular cases of this problem are widely used for benchmarking since they are challenging for numerical methods. We mention three of them. The Sod problem is a problem of strong discontinuity decay in density, pressure, and internal energy of gas moving in a shock tube [6, 7]. The numerical simulation of such a discontinuity is nontrivial and requires using of Riemann solvers or artificial viscosity. The Sjogreen problem is a problem of the motion of two diverging rarefaction waves [8]. It is difficult to compute the medium macroparameters in the low-density domain; a numerical correction of gas pressure is used to solve it. The Noh problem is a problem of the oncoming motion of supersonic flows [9]. It simulates a rapid conversion of kinetic energy to internal energy. Modeling such a transition is sensitive to artificial or scheme viscosity and often requires the addition of artificial thermal conductivity.

It is known that the ideas that make it possible to construct methods with certain properties (for example, monotonicity) in the one-dimensional case cannot always be extended to two-dimensional and three-dimensional models. Therefore, it is necessary to be able to test how the methods can be used in strong discontinuities, regions of rarefaction and rapid heating in multidimensional cases. For these purposes, the solutions of the following three-dimensional axisymmetric problems are used as reference ones: the Noh problem (see the generalized solution of the Noh problem in [10] and citations there), the Sedov's point explosion problem [11–14], the Guderley problem of the imploding shock wave [15–17]. These three-dimensional problems allow determining the requirements for the numerical resolution for reproducing the internal features of a flow in a three-dimensional case, and quantitative estimating the spherical symmetry of the solution, as well as the performance of a numerical algorithm.

The above one-dimensional and three-dimensional problems are formulated for an infinite domain filled with matter, and their numerical solution is obtained in a limited domain and requires the introduction of artificial boundary conditions [18,19] (also notated as buffer domain, absorbing layer, sponge layer, fringe domain). Apart from open boundary conditions as well as conditions on the wall (no-slip and slip) for some applications (for example, simulation of combustion and explosion), it is necessary to implement free boundary conditions (a sharp interphase boundary condition in the case of immiscible media). In this case, the pressure is continuous at the interface, the density can have a strong dis-continuity, and the position of the interface changes with time. Modeling of the moving interface is nontrivial for both Eulerian and Lagrangian methods; therefore, problems, in which such an interface is described analytically, are of interest.

The Shemarulin problem can be used as a one-dimensional test problem [20]. In this problem, the expansion of a clump of quiet gas into vacuum are considered. The gas is located in the domain $-1<x<1$ with a special initial density distribution $\rho(x,t=0)=1-x^2$