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A Physics-Wise Splitting Preconditioner with Selective Relaxation for the Multi-Group Radiation Diffusion Equations in Three Dimensions

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Abstract. Designing a good preconditioner for accelerating the iterative solution of the three-dimensional multi-group radiation diffusion equations based on a cell-centered finite volume discretization has been the focus of intensive research efforts over the past few decades. In the present paper, we develop a physics-wise splitting preconditioning algorithm with selective relaxation and algebraic multigrid subsolves. The spectral distribution and the degree of the minimal polynomial of its right-preconditioned matrix together with the conditional convergence property of its iteration method are analyzed. Subsequently, we discuss its sequential implementation as well as the two-level parallelization. Lastly, the new preconditioner is applied to the experimental test cases arising from realistic simulations of the hydrodynamic instability during the deceleration phase of a laser-driven spherical implosion to illustrate the numerical robustness, computational efficiency, parallel strong and weak scalabilities, and its competitiveness with some existing monolithic and block preconditioning approaches.

AMS subject classifications: 65F10, 65N55, 65Z05

Key words: Radiation diffusion equations, physics-wise splitting, selective relaxation, algebraic multigrid, parallel and distributed computing.

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1 Introduction

The thermal radiation transport (TRT) equations, which compactly describe the propagations of high-energy photons in a physical system and the interactions with electrons directly and ions indirectly within this system, appear in various branches of physics, such as the optical remote sensing, the massive star formation and inertial confinement fusion experiments. In the past several decades, tremendous efforts have been focused upon developing mathematically less complicated and computationally cheaper yet numerically more accurate approximations to the spatio-temporal orientation- and frequency-dependent TRT equations, among which the simplest and most extensively applied one is the flux-limited multi-group radiation diffusion (MGD) equations in a background medium with numerous materials [15], where each physical variable at a point is an integral over contributions from all orientations, and the frequency-dependent radiation energy densities are classified into a finite number of energy groups and assumed to be evenly distributed over each frequency range.

It should be emphasized that the numerical solution of the MGD equations is challenging as a result of the highly nonlinear and strongly discontinuous properties in numerous coefficients, together with the multiple spatio-temporal scales and wave-like propagation characteristics in solutions for all times after the initial time [5]. Traditionally, the adaptive backward Eulerian time-integration can be utilized to eliminate the need of severe time step constraints for stability. For the resulting nonlinear reaction-diffusion systems, we take advantage of the method of frozen coefficients [20] as the iterative linearization technique, followed by a cell-centered finite volume discretization scheme ensuring a local conservation property, however, requiring the solutions of many sparse, ill-conditioned, unsymmetric but positive definite linear systems of equations with the number of degrees of freedom ranging from 10^7 to 10^{11} as a result of the presence of hydrodynamic instabilities, which is computationally expensive, generally accounting for more than eighty percent of the total simulation time. Therefore, this motivates efforts aimed at the development of robust and accurate numerical solution algorithms in an efficient and scalable manner.

Despite their intrinsic appeals (e.g., reliability and accessibility) for smaller systems of linear equations, sparse direct solvers (e.g., MUMPS [1], PARDISO [28], PaStiX [16], STRUMPACK [13], SuperLU [21], SuperMF [31] and UMFPACK [10]) have several disadvantages, such as the considerable amount of memory footprint and difficulties in developing effective massively parallel implementations, which heavily restrict the scope of typical applications. On the contrary, sparse iterative solvers, which generate a sequence of approximate solutions and essentially involve only matrix-vector multiplications and vector-vector manipulations, become increasingly attractive to the sustaining demand for higher-spatial resolutions, due to their fairly smaller memory utilization and higher degree of parallelism. The most popular sparse iterative solvers are Krylov subspace methods, like the biconjugate gradient stabilized (Bi-CGSTAB) method [30] and the generalized minimal residual (GMRES) method [27] as two prominent representa-