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A Modified Characteristic-Wise WENO Reconstruction for Capturing the Contact Discontinuity in Multi-Component Flows

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Abstract. The flux difference splitting (FDS) combined with the characteristic-wise weighted essentially non-oscillatory (WENO) reconstruction in single-component flow simulations is quite attractive and preferred for its better non-oscillatory feature near discontinuities. However, when compressible multi-component flows are simulated, the FDS with the characteristic-wise WENO reconstruction may cause spurious oscillations. In this work, we analyze the reason that the oscillations are generated. A new method, which can effectively suppress this kind of spurious oscillations in multi-component flows, is proposed by modifying the total energy in the process of the WENO reconstruction. For example, in the fifth-order WENO reconstruction, on the global stencil $S^5=(j-2,j-1,\cdots,j+2)$ used to reconstruct the left-side value $U^L_{j+1/2}$, the total energy values are first modified by using the reconstructed value of the specific heat ratio (i.e., $\gamma^L_{j+1/2}$, which can be directly reconstructed from the transport equation of the mass fraction), and then the characteristic-wise WENO reconstruction process is implemented. Numerical examples including several one-, two and three-dimensional multi-component flows confirm the effectiveness and robustness of the proposed method.

AMS subject classifications: 65N50, 76M12, 76T20, 82C40

Key words: Contact discontinuity, flux difference splitting method, WENO scheme, characteristic-wise reconstruction, multi-component flow.

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1 Introduction

Contact discontinuities and material-interfaces are important characteristics of compressible multi-component flows. The weighted essentially non-oscillatory (WENO) scheme has the ENO property for capturing discontinuities and can keep a high-order accuracy in smooth regions, and hence has also been used to capture contact discontinuities and material interfaces [1–6]. However, the characteristic of the contact discontinuity or the material-interface is different from that of the shock wave. For example, in the Euler system of gas dynamics, the contact discontinuity means that the velocity and pressure are identically constant, while the density undertakes a jump. In addition, based on the wave-propagation theory, the local characteristic fields of compressible Euler equations can be decomposed into genuinely nonlinear and linearly degenerate fields [7]. For the Riemann problem [8], the linearly degenerate field is associated with a contact discontinuity, the genuinely nonlinear fields are associated with shocks and rarefaction waves. Hence, a good shock-capturing method may not be a valid choice for resolving a contact discontinuity directly. Johnsen [9] showed that, when a component-wise WENO reconstruction is used to compute isolated contact discontinuities in a single-component fluid, small numerical errors (in the velocity and pressure) are induced. As a result, velocity and pressure oscillations are generated. Johnsen [9] suggested applying the reconstruction of the primitive variables or characteristic variables to overcome these errors. However, such a treatment still encounters spurious numerical issues if it is directly applied to solve compressible multi-component flows.

Many studies [10–16] showed that, for multi-component flows, the conventional conservative schemes used to solve the conservative governing equations generate spurious pressure oscillations at the flow interfaces. Such oscillations often lead to numerical instabilities [17,18]. To suppress these spurious oscillations, Abgrall [11] proposed a pressure-equilibrium preserving scheme that solves the transport equation of the ratio of specific heats function instead of the conservation equation of the mass of species. Shyue [13] extended the method to the mass fraction form. Recently, several spatial discretization methods were proposed, such as the kinetic energy preserving schemes [19,20], entropy conserving schemes [21,22], the kinetic energy and entropy preserving schemes [23,24], and the pressure-equilibrium preserving schemes [16,25]. Fujimara et al. [16] proposed a pressure-equilibrium preserving method for solving the fully conservation equations by using the half-point values of the mass of species to calculate the numerical fluxes. The method in [16] is second-order accurate and effective for inviscid smooth interfaces advection problems.

Weighted essentially non-oscillatory (WENO) schemes [26–31] have a strong capability of capturing shock waves, however, when they are used to resolve the contact discontinuities, especially the interfaces of multi-component flows, the spurious oscillations still occur. Johnsen and Colonius [1] analyzed the finite volume method with the high-order WENO reconstruction, they concluded that the primitive variables must be reconstructed rather than the conservative variables. The same conclusion was also drawn in