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The Discrete Direct Deconvolution Model in the Large Eddy Simulation of Turbulence

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Abstract. The discrete direct deconvolution model (D3M) is developed for the large-eddy simulation (LES) of turbulence. The D3M is a discrete approximation of previous direct deconvolution model studied by Chang *et al.* ["The effect of sub-filter scale dynamics in large eddy simulation of turbulence," Phys. Fluids 34, 095104 (2022)]. For the first type model D3M-1, the original Gaussian and Helmholtz filters are approximated by local discrete formulation of different orders, and direct inverse of the discrete filter is applied to reconstruct the unfiltered flow field. The inverse of original Gaussian and Helmholtz filters can be also approximated by local discrete formulation, leading to a fully local model D3M-2. Compared to traditional models including the dynamic Smagorinsky model (DSM) and the dynamic mixed model (DMM), the D3M-1 and D3M-2 exhibit much larger correlation coefficients and smaller relative errors in the *a priori* studies. In the *a posteriori* validations, both D3M-1 and D3M-2 can accurately predict turbulence statistics, including velocity spectra, probability density functions (PDFs) of sub-filter scale (SFS) stresses and SFS energy flux, as well as time-evolving kinetic energy spectra, momentum thickness, and Reynolds stresses in turbulent mixing layer. D3M-1 and D3M-2 have more advantages in predicting the SFS statistics compared to scale-similarity model (SSM), DSM, and DMM. Thus, the D3M holds potential as an effective SFS modeling approach in turbulence simulations.

AMS subject classifications: 76F65, 76F05, 76F10

Key words: Direct deconvolution model, sub-filter scale model, large-eddy simulation.

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1 Introduction

Large eddy simulation (LES) is an important method for studying turbulence. LES separates large-scale and small-scale motions in turbulence through filtering operations, which solves the large-scale motions directly, and models the effects of small-scale flow structures. This allows more efficient simulation of turbulent flows with limited computational resources, especially for those flow phenomena that are mainly dependent of large-scale motions. One of the prominent challenges in LES is the accurate reconstruction of sub-filter scale (SFS) stresses [69, 74]. Over the past several decades, various SFS models have been developed [62], including the Smagorinsky model [89], dynamic Smagorinsky model (DSM) [52], and dynamic mixed model (DMM) [100, 123]. Additionally, implicit LES [13, 37, 97], which does not require explicit SFS modeling but relies on numerical dissipation to capture SFS effects, has emerged as an alternative approach. With the advancement of machine learning, artificial-neural-network-based LES methods have also gained prominence [5, 10, 14, 31, 33, 38, 44, 45, 51, 67, 68, 84, 95, 110, 112–116, 126].

In LES, filtering operation separates different scales of motion in turbulence, which helps better understand the nature of turbulence and provides more efficient simulation tools for engineering flow and fluid dynamics research [69, 74]. [91] showed that the SFS stresses can be approximately reconstructed by iteratively inverting the filtered flow field for an invertible filter. Based on this observation, the approximate deconvolution model (ADM) has been proposed and applied in the incompressible wall-bounded flows [92] and the shock-turbulent-boundary-layer interaction [93]. The ADM has successful applications in various domains, including the LES of Burgers' turbulence [4], turbulent channel flows [85], oceanography [77, 79], magnetohydrodynamics [46], combustion [25–27, 56, 60, 65, 66, 86, 106, 107], and multiphase flow [73, 82, 83]. Simulation frameworks based on deconvolution have also been adapted for temporal regularization rather than spatial regularization [70], and have also found applications in Lattice-Boltzmann methods [63]. Mathematical proofs and dedicated literature have also been developed regarding the ADM [11, 28–30, 47–49].

The approximate deconvolution model is primarily based on the van Cittert iteration [80, 81, 96]. On the basis of ADM, data-driven deconvolution methods have been developed [22, 53, 57–59]. The neural networks mapping the filtered and unfiltered fields have been established and applied in various turbulence studies [57–59]. A deconvolutional artificial neural network (DANN) model has been proposed [119, 122], where artificial neural network is used to approximate the inverse of the filter. The DANN method has also been extended to model the SFS terms in LES of compressible turbulence with exothermic chemical reactions [94]. To address the challenge of neural networks relying on the *a priori* flow field data [120], further introduced the dynamic iterative approximate deconvolution (DIAD) model, which has been applied to decaying compressible turbulence [125] and dense gas turbulence [124].

The selection of filters in LES is also crucial. [36] derived analytical expressions for inverting the box filter and utilized these expressions to develop generalized scale-similarity