Commun. Math. Anal. Appl.
doi: 10.4208/cmaa.2024-0016

Vol. **3**, No. 3, pp. 384-401
August 2024

A Review on Two Types of Sonic Interfaces

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Received 9 May 2024; Accepted 23 June 2024

Abstract. In this paper, two examples of sonic interfaces ([2–6]) are presented. The first example shows the case of sonic interfaces as weak discontinuities in self-similar shock configurations of unsteady Euler system. The second example shows the case of sonic interfaces as regular interfaces in accelerating transonic flows governed by the steady Euler-Poisson system with self-generated electric forces. And, we discuss analytic differences of the two examples, and introduce an open problem on decelerating transonic solution to the steady Euler-Poisson system.

AMS subject classifications: 35C06, 35M10, 35M30, 35Q31, 76H05, 76N10

Key words: Keldysh type, transonic, sonic interface,weak discontinuity, regular interface.

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1 A sonic interface as a weak discontinuity

Fix a constant $\varepsilon_0 > 0$. Given a function $f:[0,\varepsilon_0] \to \mathbb{R}_+$ with

$$\begin{aligned} &\|f\|_{C^{1,1}([0,\varepsilon_0])} < \infty, \quad && f(0) > 0, \\ &\frac{df}{dx} \geq \omega > 0, && \forall 0 \leq x \leq \varepsilon_0, \end{aligned} \tag{1.1}$$

set

$$P_0 := \big(0, f(0)\big),$$

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and define the domain

$$\mathcal{Q}^f_{\varepsilon_0} := \{(x,y) : 0 < x < \varepsilon_0, 0 < y < f(x)\}. \tag{1.2}$$

For each $t \in (0, f(0))$, set

$$\mathcal{R}_t := \left(0, \frac{\varepsilon_0}{2}\right) \times \left(0, f(0) - t\right).$$

Given two constants $a > 0$ and $b > 0$, and functions $\beta_k \in C(\partial \mathcal{Q}^f_{\varepsilon_0} \cap \{y = f(x)\})$ for $k = 1,2,3$, consider the equation

$$(2x - a\psi_x + O_1)\psi_{xx} + O_2\psi_{xy} + (b + O_3)\psi_{yy} - (1 + O_4)\psi_x + O_5\psi_y = 0 \quad \text{in } \mathcal{Q}^f_{\varepsilon_0}, \tag{1.3}$$

and the boundary conditions

$$\psi = 0 \qquad \text{on } \partial \mathcal{Q}^f_{\varepsilon_0} \cap \{x = 0\}, \tag{1.4}$$

$$\partial_y \psi = 0 \qquad \text{on } \partial \mathcal{Q}^f_{\varepsilon_0} \cap \{y = 0\}, \tag{1.5}$$

$$\beta_1(x,y)\psi_x + \beta_2(x,y)\psi_y + \beta_3(x,y)\psi = 0 \quad \text{on } \partial \mathcal{Q}^f_{\varepsilon_0} \cap \{y = f(x)\}. \tag{1.6}$$

In addition, assume that

$$\beta_1(x,y) \geq \lambda, \quad |\beta_2(x,y), \beta_3(x,y)| \leq \frac{1}{\lambda} \quad \text{on } \partial \mathcal{Q}^f_{\varepsilon_0} \cap \{y = f(x)\} \tag{1.7}$$

for some constant $\lambda > 0$.

Theorem 1.1 ([1, Theorems 3.1, 4.2]). *Suppose that a function $\psi : \overline{\mathcal{Q}^f_{\varepsilon_0}} \to \mathbb{R}$ satisfies the following conditions:*

(i) $\psi \in C^2(\mathcal{Q}^f_{\varepsilon_0}) \cap C^{1,1}(\overline{\mathcal{Q}^f_{\varepsilon_0}})$,

(ii) $\psi > 0$ *in* $\mathcal{Q}^f_{\varepsilon_0}$,

(iii) there exist constants $\mu > 0$ and $\delta \in (0,1)$ such that

$$-\mu \leq \frac{\psi_x(x,y)}{x} \leq \frac{2-\delta}{a} \quad \text{in } \mathcal{Q}^f_{\varepsilon_0},$$

(iv) ψ *satisfies* (1.3)-(1.6).