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An Existence Result for a Mathematical Model of Koiter's Type

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Abstract. In this paper we first introduce a new shell model that can be applied for all kinds of geometries of the middle surface of the shell. Then we show that our model is close to Koiter's nonlinear shell model in a specific sense. Finally, we establish the existence of a minimizer for this new model.

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1 Introduction

A nonlinearly elastic shell with constant thickness is a three-dimensional elastic body whose reference configuration consists of all points that lie within a small given distance from a given surface, which is called the "middle surface of the shell". The nonlinear Koiter's shell model, introduced by Koiter (see [9]) in 1966, is one of the most used two-dimensional nonlinearly elastic shell models in numerical simulations. It states that the unknown deformation $\boldsymbol{\varphi}:\omega\to\mathbb{R}^3$ of the middle surface $S=\boldsymbol{\theta}(\overline{\omega})$ of the shell subjected to applied forces should minimize a functional

$$J_K(\boldsymbol{\varphi}):=\int_\omega W_K(\boldsymbol{\varphi})\sqrt{a}\,dy-L_K(\boldsymbol{\varphi}), \tag{1.1}$$

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called the total energy of the deformed shell, over an appropriate set of admissible deformations. Here W_K denotes Koiter's stored energy function (which will be defined later) and L_K denotes a linear form that takes into account the applied forces. However, as far as we know, no theorem has been established in the literature proving the existence of a such minimizer.

On the other hand, several existence theorems have been established for ad hoc approximations of Koiter's shell model, that is, for models whereby Koiter's stored energy function $W_K(\boldsymbol{\varphi})$ is in (1.1) replaced by

$$\widetilde{W}_K(\boldsymbol{\varphi}) := W_K(\boldsymbol{\varphi}) + \mathcal{R}(\varepsilon,\boldsymbol{\varphi}),$$

where the additional term $\mathcal{R}(\varepsilon,\boldsymbol{\varphi})$ is negligible compared with $W_K(\boldsymbol{\varphi})$ in some meaningful sense. Bunoiu *et al.* [3] and Ciarlet and Mardare [5] proposed a well-posed two-dimensional approximation of Koiter's model for spherical and "almost spherical" shells. Giang and Mardare [8] established existence theorems for nonlinear shell models asymptotically equivalent to Koiter's model for the shells whose middle surfaces are minimal surfaces. Finally, Anicic [1,2] proposed an approximate model of Koiter's model that has a minimizer over the set of deformations whose principal radii of curvatures are bounded below by the half thickness of the shell. A different approach by Ciarlet and Mardare [6] and Mardare [10], where the authors have proposed nonlinear shell models asymptotically equivalent to the Koiter's model for all kinds of geometries, but depending on the transverse variable, so that these are three-dimensional models.

The purpose of this paper is to define a well-posed two-dimensional shell model that is approximately equivalent to that of Koiter without any restrictions on the geometry of the middle surface of the shell. Our approach is similar to that of Anicic [1,2], the difference being that in our model the space of admissible deformations is independent of the thickness of the shell. The definition of our model is based on the ideas first appearing in the papers of Giang and Mardare [8] and Anicic [1,2].

2 Notations and definitions

In all that follows, Greek indices and exponents range in the set $\{1,2\}$ while Latin indices and exponents range in the set $\{1,2,3\}$ (except when they are used for indexing sequences). The Einstein summation convention with respect to repeated indices and exponents is used.

Vector and matrix fields are denoted by boldface letters. The Euclidean norm, the inner product and the vector product of two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in $\mathbb{R}^3$ are respectively denoted $|\boldsymbol{u}|, \boldsymbol{u}\cdot\boldsymbol{v}$ and $\boldsymbol{u}\wedge\boldsymbol{v}$. Given any integers $m \geq 1$ and $n \geq 1$, the inner