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Decay Structure of the Zener-Type Viscoelastic Plate with Type II Heat Conduction in the Whole Space

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Abstract. In this paper, we study the Zener-type viscoelastic plate in that heat conduction is determined by the type II Green-Naghdi theory. This model considers the finite propagation of thermal waves, which is compatible with the causality principle. We consider the Cauchy problem of this model and derive the decay estimates of solutions. Furthermore, the optimality of the decay estimate is discussed.

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1 Basic equations

The knowledge of thermoelastic plates has recently merited extensive study [1,4–7]. However, many of the models used allow the instantaneous propagation

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of mechanical and/or thermal waves, which is not compatible with the principle of causality. Indeed, Kelvin-Voigt viscoelasticity has been widely used and studied, but it is known that it suffers from the effect mentioned above. The same can be said of Fourier-type heat conduction (or Green-Naghdi type III).

On the other hand, Zener-type viscoelasticity [8] proposes a hyperbolic equation to describe the deformations and Green-Naghdi type II heat conduction [3] also proposes a hyperbolic equation to describe the heat conduction. In this article we will consider a Zener-type viscoelastic plate with Green-Naghdi type II heat conduction, therefore a model compatible with the principle of causality.

It is worth noting that in most of the studies discussed above, a conservative structure for the mechanical component is combined with a dissipative one for heat conduction. In this article, we want to study a different situation when the structure is dissipative for the mechanical part and conservative for heat conduction. In summary, in this article we are going to study a plate model fully compatible with the principle of causality whose mechanical part is dissipative and the thermal part is conservative and which occupies the entire n-dimensional space. We wish to clarify the asymptotic behavior and the regularity of the solutions to the Cauchy problem. We will therefore study the counterpart of the problem studied in [2] in the case where the plate occupies the entire space.

The system of the Zener-type viscoelastic plate with type II heat conduction is described as follows (see [2]):

$$\begin{aligned} &\tau\rho u_{ttt}+\rho u_{tt}+\mu\Delta^2 u+\mu^*\Delta^2 u_t+\beta\Delta\alpha_t=0,\\ &\gamma\alpha_{tt}-\kappa\Delta\alpha-\beta\Delta u_t-\tau\beta\Delta u_{tt}=0 \end{aligned} \tag{1.1}$$

for $t>0$ and $x=(x_1,\cdots,x_n)\in\mathbb{R}^n$. The parameters $\tau,\rho,\mu,\mu^*,\beta,\gamma$ and κ are constitutive coefficients which satisfy $\tau,\rho,\mu,\mu^*,\gamma,\kappa>0$, $\beta\neq 0$ and $\mu^*-\tau\mu>0$. Note that the thermomechanical meaning of these assumptions is clear. In this article, we focus on the Cauchy problem defined by the system (1.1) with the initial data

$$(u,u_t,u_{tt},\alpha,\alpha_t)(0,x)=(u_0,u_1,u_2,\alpha_0,\alpha_1)(x)$$

for $x\in\mathbb{R}^n$, where u_0,u_1,u_2,α_0 and α_1 are given scalar functions. Considering (1.1) with $\tau=0$ formally, the system (1.1) is reduced to the viscoelastic plate with type II heat conduction, which is analyzed in [7].

To analyze the dissipative structure for (1.1), it is convenient to rewrite the system in the following new variables and derive the decay estimate of solutions. We introduce

$$v:=\tau u_{tt}+u_t,\quad w:=\tau\Delta u_t+\Delta u,\quad y:=\Delta u_t,\quad \theta:=\alpha_t,\quad \phi:=\nabla\alpha, \tag{1.2}$$