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On Some Cardinal Properties of the Space $P_f(X)$ of Probability Measures

Shavkat Ayupov[1,2] and Nodirbek Mamadaliev[1,2,*]

[1] *V.I. Romanovskiy Institute of Mathematics Uzbekistan, Academy of Sciences, Tashkent 100174, Uzbekistan.*

[2] *National University of Uzbekistan, Tashkent 100174, Uzbekistan.*

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Abstract. In this paper, given a compact space X, we construct a continuous mapping from the space $P_f(X)$ of all probability measures on X into the space of maximal linked systems of closed subsets of X. Also, we investigate the behavior of the functional tightness and the local density of topological spaces under the action of the functor $P_f\colon Comp \to Comp$. We show that the functor P_f does not change the functional tightness and the local density of compact spaces.

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1 Introduction

The concept of the functional tightness of topological spaces was first introduced by Arkhangelsky [1]. The action of closed and R-quotient maps on functional tightness was investigated by Okunev and Paramo [13]. They proved that R-quotient mappings do not increase the functional tightness. Also, they showed that the functional tightness of the product of two locally compact spaces does not exceed the product of functional tightness of those spaces [13]. Krupski [12] generalized the result of Okunev and Paramo as follows:

*Corresponding author. *Email addresses:* `shavkat.ayupov@mathinst.uz` (S.A. Ayupov), `nodir_88@bk.ru` (N.K. Mamadaliev)

Theorem 1.1 ([12]). *For any infinite compact space X the following equality holds:*

$$t_0(X)=t_0(X^c).$$

There are some investigations of the action of covariant functors to cardinal invariants of various classes of topological spaces. The actions of some classic covariant functors on cardinal properties of topological spaces are investigated, for example, in [2, 4–7]. In [5] the authors investigated the behavior of the minimal tightness and functional tightness of topological spaces under the action of an exponential functor of finite degree. It was proved that the exponential functor preserves the functional tightness and the minimal tightness of compact spaces. In [2] we investigated how the functor of weakly additive functionals affects the minitightness and the tightness of topological spaces. As well as, in [2] it was shown that the minitightness and the local density of compact spaces are preserved by the functor O_n.

In [8] the local density and the local weak density of topological spaces were introduced and the following results for exponential functor $\exp_n$ were obtained.

Theorem 1.2 ([8]). *For every infinite T_1-space X the following equalities hold:*

1. $ld(X)=ld(exp_n(X))$;
2. $lwd(X)=lwd(exp_n(X))$.

The functor $P_f: Comp\to Comp$ was introduced in [14]. Some properties of this functor were investigated by several authors in recent years. Zaitov [16] showed that for a compact X the space $P_f(X)$ is an absolute neighborhood retract if and only if X is an absolute neighborhood retract. It is proved that the functor P_f preserves the property of being Q-manifold or Hilbert Cube, preserves the property of fibers of mappings to be an absolute neighborhood retract in the class of compact spaces. Further, in [17] some topological properties of the functor P_f were investigated and it was proved that the space $P_f(X)$ is weakly countable dimensional, if so is X.

In this paper, we construct a continuous mapping from the space $P_f(X)$ into the space of maximal linked systems of closed subspaces in X. Also, we investigate the behavior of the functional tightness and the local density of topological spaces under the action of the functor $P_f: Comp\to Comp$. We prove that the functor P_f does not change the functional tightness and the local density of compact spaces.

For main definitions we basically refer to [10, 11]. Throughout the paper, all spaces are assumed to be completely regular and Hausdorff, τ means an infinite cardinal number, the cardinality of a countable set is denoted by ω.