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Uniqueness of Transonic Shock Solutions to Euler-Poisson System with Varying Background Charges

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Abstract. In this paper, we investigate a one-dimensional Euler-Poisson system with varying background charges, which are two different constants when the flow speed is less than and greater than the sound speed. Using the shock matching method, we derive the properties of the solution trajectories and establish a monotonic relationship between the density at the nozzle exit and the shock position. This relationship demonstrates the existence and uniqueness of a transonic shock solution under suitable boundary conditions.

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Key words: Euler-Poisson system, transonic shock solution, uniqueness, varying background charges.

1 Introduction

The hydrodynamical model for the motion of electrons in semiconductor devices or plasmas is generally governed by the following Euler-Poisson system, which consists of equations describing the conservation of mass and momentum, coupled with Poisson's equation for the electric field:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (p(\rho) + \rho u^2)_x = \rho E, \\ E_x = \rho - b, \end{cases} \tag{1.1}$$

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where u,ρ and p represent the average particle velocity, density and pressure, respectively. E denotes the electric field which is generated by the Coulomb force of particles, and the function $b>0$ stands for the density of positively charged background ions [17]. Assume that p satisfies

$$p(0)=0, \quad p'(\rho)>0, \quad p''(\rho)>0 \quad \text{for} \quad \rho>0, \quad p(+\infty)=+\infty. \tag{1.2}$$

In the biological field, the system can also be used to simulate the transport of ions between the extracellular side and the cytoplasmic side of the membranes [7]. In this case, ρ, ρu, and E are the ion concentration, ion translational mass, and electric field, respectively.

In this study, we focus on the steady case for (1.1) with varying background charges. These charges are represented as two different constants when the flow speed is less than and greater than the sound speed, respectively. The boundary value problem is written as

$$\begin{cases} (\rho u)_x=0, \\ (p(\rho)+\rho u^2)_x=\rho E, \\ E_x=\rho-b \end{cases} \tag{1.3}$$

with the boundary value conditions

$$(\rho,u,E)(0)=(\rho_0,u_0,E_0), \quad (\rho,u)(L)=(\rho_e,u_e), \tag{1.4}$$

where ρ_0,u_0,E_0,ρ_e and u_e are given positive constants. We assume that b is a piecewise constant function of the form

$$b=\begin{cases} b_1, & u>c, \\ b_2, & u<c, \end{cases} \tag{1.5}$$

where b_1 and b_2 are positive constants, and, by the terminology of gas dynamics, $c=\sqrt{p'(\rho)}$ represents the sound speed.

The first equation in (1.3) indicates that ρu is a constant and we denote it by J. Thus the boundary value problem (1.3)-(1.4) can be further reduced to the following ODE system for (ρ,E):

$$\begin{cases} \rho_x=\dfrac{\rho E}{p'(\rho)-J^2/\rho^2}, \\ E_x=\rho-b \end{cases} \tag{1.6}$$

with the boundary conditions

$$(\rho,E)(0)=(\rho_0,E_0), \quad \rho(L)=\rho_e. \tag{1.7}$$