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Functional Solutions, Multi Line Solitons and Multiple Pole Solutions of the Generalized (2+1)-Dimensional Kaup-Kupershmidt Equation

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Abstract. The (2+1)-dimensional integrable generalization of the Kaup-Kupershmidt (KK) equation is solved by the inverse spectral transform method in this paper. Several new long derivative operators V_x, V_y and V_t and the kernel functions K of $\bar{\partial}$-problem are introduced to construct a type of general solution of the KK equation. Based on these, several classes of the new exact solutions, with constant asymptotic values at infinity $u|_{x^2+y^2\to\infty}\to 0$, for the KK equation are constructed via the $\bar{\partial}$-dressing method.

AMS subject classifications: 35C08, 45G15, 45Q05

Key words: KK equation, inverse scattering transform, $\bar{\partial}$-dressing method, exact solution.

1 Introduction

The inverse scattering transform method was proposed by Gardner *et al.* [18] in 1967 to study initial value problems related to the famous Korteweg-de Vries (KdV) equation. In recent years, the inverse spectral transform method [27] has been generalized and successfully applied to the computation of a wide class of

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exact solutions of various (2+1)-dimensional nonlinear evolution equations. For example, the works [1, 2, 10, 11, 17, 22, 26, 28, 32]. The basic tools for solving nonlinear equations via inverse scattering transform are now the non-local Riemann-Hilbert problem [12], the $\bar{\partial}$-problem [5] and the $\bar{\partial}$-dressing method [3,4]. Different kinds of exact solutions of some (2+1)-dimensional integrable nonlinear evolution equations were studied by the $\bar{\partial}$-dressing method [6, 8, 9, 13, 24]. Such as the line solitons and line rational lumps of the Savada-Kotera (SK) equation [15], the multiple pole solutions of the Kadomtsev-Petviashvili (KP) equation, the modified Kadomtsev-Petviashvili (mKP) equation and the Davey-Stewartson (DS) system of equations [14], the plane solitons of mKP equation [20], the rational solutions of the generalizations of dispersive long wave, nonlinear Schrödinger, sinh-Gordon and heat equations [13]. The study of exact solutions of integrable nonlinear equations, such as soliton solutions, multi-pole solutions, rational solutions, etc. is an important topic in the field of mathematical physics. The $\bar{\partial}$-dressing method is a very convenient and powerful method for computing exact solutions of (2+1)-dimensional nonlinear equations.

In this paper, we construct solutions with functional parameters, multi line solitons solutions and multiple pole solutions of the KK equation (see [21])

$$\begin{aligned} u_t = & u_{xxxxx} + 5uu_{xxx} + \frac{25}{2}u_x u_{xx} + 5u_x \partial_x^{-1} u_y \\ & -5\partial_x^{-1} u_{yy} + 5u_{xxy} + 5uu_y + 5u^2 u_x, \end{aligned} \tag{1.1}$$

which has important application in fluid mechanics and plasma physics, where u depends on time variable t and space variables x,y. The line solitons and line rational lumps of Eq. (1.1) have been studied. At the same time, we need know that ∂_x^{-1} is an inverse operator of ∂_x, $\partial_x^{-1}\partial_x = \partial_x\partial_x^{-1} = 1$. For $\partial_y = 0$, Eq. (1.1) reduces to the (1+1)-dimensional KK equation which was considered by Kaup and Kupershmidt.

Eq. (1.1) possesses the following operators:

$$\begin{aligned} L\Phi = & \Phi_y + \Phi_x^3 + u\Phi_x + \frac{1}{2}u_x\Phi = 0, \\ T\Phi = & \Phi_t - 9\Phi_x^5 - 15u\Phi_x^3 - \frac{45}{2}u_x\Phi_x^2 \\ & -\left(\frac{35}{2}u_{xx} - 5\partial_x^{-1}u_y + 5u^2\right)\Phi_x \\ & -\left(5u_{xxx} + 5uu_x - \frac{5}{2}u_y\right)\Phi = 0. \end{aligned} \tag{1.2}$$