Commun. Math. Res.
doi: 10.4208/cmr.2025-0007

Vol. **41**, No. 2, pp. 189-208
June 2025

Uniqueness in Inverse Electromagnetic Scattering by Locally Rough Surfaces with Phaseless Near-Field Data

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Received 11 February 2025; Accepted 7 June 2025

Abstract. In this paper, we consider the reconstruction of a locally rough surface from phaseless near-field data generated by the incident electric dipoles. To obtain the uniqueness for this inverse problem, we use the superposition of point sources as the incident waves. These point sources lie on an admissible surface. The measured data are collected from another admissible surface above this locally rough surface. We derive the Rellich's lemma and the reciprocity relation for the electric total fields. Based on them, we establish the uniqueness in phaseless inverse electromagnetic scattering by locally rough surfaces.

AMS subject classifications: 78A46, 35P25

Key words: Uniqueness, inverse electromagnetic scattering, phaseless near field, locally rough surface, reciprocity relation.

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1 Introduction

Direct and inverse scattering theories play a central role in many applications, such as radar, sonar, geophysical exploration, medical imaging, and non-destructive testing. They have attracted mathematicians and physicists to study them for over a hundred years. The direct scattering problem has been extensively researched, and a large number of numerical results were obtained by integral equation methods [8] and finite element methods [13,16–19]. In comparison, the inverse scattering problem, which is the forefront of mathematical research in

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scattering theory, has progressed only since the 1980s from ad hoc techniques lacking rigorous mathematical basis to a rapidly growing area of intense activity with a solid mathematical foundation. We refer to [4,6,26,31,34,38,53] for various inverse scattering results including theoretical and numerical methods.

Since it is difficult to obtain accurate full-phase information from measurements in practical applications, the use of phaseless data for inverse scattering problems is more advisable and has been attracted much attention in numerical methods, such as iteration algorithms for inverse obstacle scattering [9, 11, 14, 20, 24], inverse periodic structure diffraction [3,32], inverse cavity scattering [28] and locally/globally rough surface [5, 46], the direct sampling methods for inverse elastic scattering [15], the approximate factorization method for inverse acoustic scattering [49], the direct imaging algorithm for inverse obstacle scattering [48], locally rough surfaces [41], and buried obstacles scattering [7, 25], the Bayesian approach for inverse acoustic obstacle scattering [44], the neural network method for recovering scattering obstacles [45].

A significant challenge in solution of the phaseless inverse obstacle scattering problem is to determine the location of the obstacle from the phaseless far-field data, due to the fact that the modulus of the phaseless far-field pattern is invariant under translations. By using infinitely many sets of superpositions of two plane waves as the incident fields at a fixed frequency, the authors of [47] numerically recovered both the location and the shape of the obstacle simultaneously from multi-frequency phaseless far-field data, and theoretically proved uniqueness in [39]. Alternatively, reference balls is introduced into the inverse acoustic obstacle scattering system to remove the translation invariance property in [50] by using the superposition of a plane wave and point sources, and in [40] by using the superposition of a fixed plane wave and plane waves in any direction. Recently, in order to remove the artificial reference, the authors of [37] introduced an admissible surface to uniquely determine the location and shape of the obstacle and its boundary condition or refractive index. This method was extended to the uniqueness for inverse obstacle electromagnetic scattering problem with known superposition of incident electric dipoles by the phaseless far-field data in [33]. We also refer to [42,51] for the uniqueness in inverse acoustic and electromagnetic scattering with phaseless near-field data at a fixed frequency.

The locally rough surface is a locally perturbed half-plane, which is contained in a planar surface over a finite interval. Theoretical and numerical results for direct scattering by such structure have been widely studied [1,2,13,27,35,43,52]. For the inverse problem of reconstructing the locally rough surface from full phase data, there has been a lot of work; see [21, 30, 35, 52] for Newton iteration methods and [10, 22, 23] for the linear sampling method. In the present paper,