

A Mathematical Analysis for the Dynamics of Multiple Languages

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Received 1 December 2024; Accepted 5 April 2025

Abstract. In many regions of the world, languages coexist in daily life, but often one tongue increases its use at the expense of another. In the present paper, we build a large compartmental system of differential equations that meets the situation of two “prestigious” tongues and many local languages, whose use is reduced by social interaction. The focus is on the preferred language in social relationships for communicating, rather than mere knowledge. We aim at stating and proving theorems on the qualitative behavior of the system. Numerical simulations illustrate the results, giving rise to distinct dynamics.

AMS subject classifications: 34A34, 37M05, 91D10

Key words: Language competition, mathematical model, dynamical system, simulation.

1 Introduction

There is no unique theory on the origin of languages, neither spoken nor signed. Two examples are the “continuity theories” and “discontinuity theories”. In the first case, the main idea is that language is so complex that it cannot appear from nothing, and therefore it is supposed that language evolves from some prelinguistic systems used by primate ancestors. This position is defended by Ulbaek [31], who argues that language evolves from primate cognition. In the second case, language is supposed to appear as a single chance mutation, as suggested by Chomsky [6], dated about 100,000 years ago. On the other hand, the “usage-based theory of language acquisition” was introduced by Tomasello [30], who states that language structure appears from language use, because tongue acquisition is done with general cognitive processes.

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It was only quite recently that UNESCO started to consider preserving Intangible Cultural Heritage. The text of the convention [32] was approved during the General Conference held in Paris in 2003. This convention divides the Intangible Cultural Heritage into five domains, being the first one “Oral traditions and expressions, including language as a vehicle of the Intangible Cultural Heritage”. The key point of this domain refers to language being a vehicle of transmission of culture. Following [33], language as culture also has three important aspects, namely: a) culture is a product of history which it in turn reflects; b) the second aspect of language as culture is as an image-forming agent in the mind of a child; and c) culture transmits or imparts those images of the world and reality through the spoken and written language, that is, through a specific language.

The implementation of foreign languages has occurred all over the world and can be studied from many points of view. The main aim of our work is to mathematically analyze the influence of two major languages (understood as “prestigious” by high society) into a community using a number J of their own local languages, by employing abstract compartmental models of ordinary differential equations and dynamical-systems theory.

A compartmental system of ordinary differential equations models a continuous-time phenomenon, such as that of language shift within a community, by analyzing rates of change and fluxes of individuals between groups. It is built on a macroscopic framework, where processes are studied from an aggregate and averaged point of view. The prototypical compartmental model is the SIR formulation, first developed in 1927 [14]. In the simplest case, it divides a population suffering a contagious disease into susceptible (S), infected (I), and recovered (R) persons, and fluxes represent contagions and recoveries [4, 34]. The output is not at individual level, but a mean value for the region (for which a probabilistic interval could be incorporated if stochastic ordinary differential equations were used). Extensions of the SIR equations have been utilized for different diseases, such as HIV/AIDS, COVID-19, etc. [23, 27, 28]. However, some of these types of models are not restricted to epidemics. Social interactions do not only transmit viruses and bacteria, but serve for spreading opinions, habits, ideologies, etc. [3, 11, 15, 29]. Since tongues and attitudes on them can be “transmitted” by contacts, some compartmental models should be useful tools to capture the mechanisms of language acquisition. These ideas have been used for various social phenomena, such as drug consumption [35], criminality [10, 18], and telecommunications [5].

The seminal one-page paper [1] initiated the work on the mathematical modeling of language interaction through ordinary differential equations and dynamical systems. The authors proposed a planar system to study the coexistence of two monolingual groups in a region, with contacts represented by probabilities of shift and with a parameter of language’s status. Some data were fitted for Scottish Gaelic, Quechua, and Welsh. Later, reference [19] extended the theory to three equations, giving the possibility of stable bilingualism. Data on Castilian and Galician in Spain were fitted. Other alternative developments were given in [25], where spatial diffusion was added to the model of [1] to situate the tongues in different geographical areas, and in [26], where a logistic natural growth of languages was suggested. In Spain, the Galician language was further