CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2023-0002

Vol. **6**, No. 1, pp. 96-105
March 2025

Economic ProductRank and Quantum Wave Probability

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Received 19 January 2023; Accepted 7 November 2024

Abstract. This note discusses a long debated question whether there is randomness in quantum mechanics or not? Einstein's view on the question is "God does not throw dice". Our starting point for the discussion is the classification of products in the economic system, called ProductRank, which seems an analog of the "principal component analysis" in statistics. But the former is much more elaborate than the latter. Interestingly, we find an intrinsic common point among economic system, statistics and quantum mechanics, which then leads to a successful classification of the products in economy, as well as a mathematical view of "wave probability" in quantum mechanics. An application to the algorithm for eigenpair is included.

AMS subject classifications: 91B, 15B57, 81S

Key words: Economics, ProductRank, Hermitizable, quantum mechanics.

1 Introduction

The problem mentioned above was motivated from Born's suggestion (1926) saying that the Schrödinger's wave function describes "waves of probability": "The square of the amplitude (of the wave function) represents the probability density of finding the particle in a certain place at a certain time"(cf. [9, p. 114]). Refer also to [4] for a survey and references therein. Here we introduce a mathematical view of Born's annotation based on our recent study on the classification of the products in economic system. For which an advanced probabilistic tool – Markov chains is adopted. However, as can be seen very soon that there is essentially no randomness in the story.

The main results of the note are stated in the next two sections. First on economics (Section 2), then on quantum, and finally on algorithm (Section 3). Their proofs are delayed to the last section (Section 4).

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2 Ranking the products in an economic system

Denote by $x=(x^{(1)},x^{(2)},\ldots,x^{(d)})$ the vector of products we are interested in the economic system. Then the evolution of the system is mainly determined by its structure matrix $A=(a_{ij}:i,j=1,2,\ldots,d)$ which means that to produce one unit of the i-th product, one requires a_{ij} units of the j-th product. Thus, once we have an input x_0, then the output x_1 in one year satisfies the equation $x_0=x_1A$. In general, we have $x_0=x_nA^n$ and then

$$x_n=x_0A^{-n}, \quad n\geq 1,$$

assuming that A is nonnegative, irreducible and invertible. This is the well-known input-output method.

Denote by $\rho(A)$ the maximal eigenvalue of A, the corresponding left- or right-eigenvectors are denoted by u (row) and v (column), respectively.

From the above simplest idealized model, one can already see the main points of Hua's optimization of global economic system (the result was appeared firstly in 1984, refer to [5,8] for a short history on the topic):

- For fastest growing rate of the system, the optimal solution of the initial input is $x_0=u$, for which we have $x_n=x_0\rho(A)^{-n}, n\geq 1$.
- If A has at least one positive diagonal element, then to keep x_n to be positive for each $n\geq 1$, the optimal solution (actually the only one) is again $x_0=u$.

The first assertion above is not so surprising, simply an application of the Perron-Frobenius theorem plus the min-max strategy. The second one is the main contribution of Hua, never appeared before as far as we know. It is even more serious that the system will be collapsed exponentially fast once $x_0\neq u$. Therefore, it is important to know the classification of the products in the economic system: the pillar products, the intermediate products and the disadvantaged products, since the system can often be collapsed at some disadvantaged products.

Following Google's PageRank (appeared in 1998), a natural way to ordering the products is using the maximal left-eigenvector u of A. However, since the matrix A in economy is quite far away from the matrix used in the network, where one has a nice graphic structure. Especially, it is far way to be a transition probability matrix. More seriously, the economic system is very sensitive, much more precise computations are required, and thus we should examine the corresponding ProductRank more carefully than the PageRank. Recall by Perron-Frobenius theorem, every nonnegative irreducible matrix A has three characteristics:

- Its maximal eigenvalue $\rho(A)$ is positive and simple.
- Its maximal left-eigenvector u is positive and one-dimensional.
- Its maximal right-eigenvector v is also positive and one-dimensional.