CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2022-0023

Vol. **6**, No. 2, pp. 320-349
June 2025

Well-Posedness of the Ocean-Atmosphere Coupled Model

Ruxu Lian[1,2,3,*], Jieqiong Ma[2], Jiangbo Jin[1,2] and Qingcun Zeng[2]

[1] *National Key Laboratory of Earth System Numerical Modeling and Application, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, P.R. China.*
[2] *International Center for Climate and Environment Sciences, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, P.R. China.*
[3] *School of Mathematics and Statistics, North China University of Water Resources and Electric Power, Zhengzhou 450046, P.R. China.*

Received 20 June 2022; Accepted 10 January 2025

Abstract. In this work, we consider the dynamic framework of the ocean-atmosphere (O-A) coupled model with physical boundary conditions at the ocean-atmosphere interface, and this coupled model can be viewed as atmosphere general circulation model coupled with ocean general circulation model. As the initial data and boundary conditions are assumed to meet certain assumptions, by taking advantage of energy estimates method and compactness arguments, we addressed the existence and stability of global weak solutions, the existence and uniqueness of global strong solution to the O-A coupled model.

AMS subject classifications: 35Q35, 76D03

Key words: Ocean-atmosphere coupled model, weak solutions, strong solution, well-posedness, physical boundary conditions.

1 Introduction

Developing accurate and reliable Earth system models (ESMs) is one of the most challenging issues in the field of earth science both domestically (in China) and internationally (see [41, 43]); and within that challenge, the ocean-atmosphere coupled model is not only the core component of ESMs, but also the key to determining their performance. Most studies of the ESM/O-A coupled model in recent years have focused mainly on im-

*Corresponding author. *Email address:* `lianruxu@mail.iap.ac.cn` (R. X. Lian)

 ©2025 Global-Science Press

proving the physical parameterization schemes and evaluating the simulation and prediction performance (see [8, 18, 19, 44–46]). However, there is not enough attention to investigation of the well-posedness theory of O-A coupled model. If the mathematical model used cannot be proved to be well posed in theory, the credibility of the simulation results will be a big problem, at least the uncertainty of numerical simulation will be greatly increased.

In order to better simulate the evolution and physical mechanism of each sphere of the Earth system, it is necessary to study in depth the well-posedness theory of the model employed. This research idea was also clearly put forward by Zeng [38]. Later, Zeng [39] also proposed the dynamic framework of the O-A coupled model and the physical boundary value conditions at the ocean-atmosphere interface. Some years later, Lions *et al.* [29–31] established a new form of large-scale ocean-atmosphere coupled equations and produced the existence of global weak solutions to the initial boundary value problem of this system. Based on the results mentioned above, Zeng [40] proposed a more practical atmosphere equations, that is the dynamic framework of atmosphere general circulation model (IAP-AGCM). The existence of global weak solutions and the uniqueness of global strong solution to the initial boundary value problem of this atmosphere equations have also been proven (see [34]). Moreover, Zeng and Mu [42] made improvements to the O-A coupled model, and Huang and Guo [17] also proved the existence of the attractor to the dynamic framework of IAP-AGCM.

Later, González *et al.* [9], Temam *et al.* [33], Cao and Titi [6] proved the existence of the local and global strong solution to atmosphere, ocean or ocean-atmosphere coupled equations, etc. According to the above research results, the well-posedness of the different moist atmosphere equations were further proved (see [10, 13, 15, 35–37]). Recently, Cao, Li and Titi *et al.* [1–5,7,20] study some important issues regarding the well-posedness of the primitive equations with partial viscosities or diffusivities. Moreover, Zhao, Huang and Guo [48] consider the blow-up criterion of the H^1 norm of solutions to the primitive equations with only horizontal viscosity and horizontal thermal diffusivity.

Compared with the theoretical model studied by mathematicians, the dynamic framework of the O-A coupled model of ESMs is more complex, and hence there are still few theoretical results on the well-posedness. In recent years, based on research methods used in studying primitive equations, Lian and co-authors considered the well-posedness of the dynamic framework of IAP-AGCM and ocean general circulation model (IAP-OGCM). For instants, Lian *et al.* [25, 27] proved the L^1-stability of weak solutions and the existence of the unique strong solution to the dynamic framework of IAP-AGCM, etc. Lian, Ma and Zeng [22, 23, 32] studied the existence and uniqueness of the global strong solution in the H^1 norm and the well-posedness of the unique local and global strong solution in the H^2 norm to the dynamic framework of IAP-AGCM with phase transformation of water vapor. Zhang *et al.* [47] and Lian *et al.* [28] also demonstrated the existence and stability of global weak solutions to the dynamic framework of IAP-OGCM. Moreover, there are also some relevant studies about the well-posedness of the primitive equations, we refer to [11, 12, 14, 16, 24, 26].